A sensorless PMSM drive using a two stage extended Kalman estimator

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***Abstract*—This paper presents an efficient implementation of the extended Kalman estimator used for the estimation of position and rotor velocity of a permanent magnet synchronous motor. An algorithm proposed by C.S. Hsieh and F.C. Chen in 1999 for linear parameter estimation is extended to the non-linear estimation, where parameters such as position and velocity are present in the transition matrix and in the augmented state space. Compared to a straightforward implementation of the extended Kalman estimator in a standard implementation, our modified optimal two-stage Kalman estimator reduces the number of arithmetic operations, allowing higher sampling rate, the estimation of parameters or the use of a cheaper microcontroller.**

I. Introduction

Permanent magnet synchronous motors (PMSMs) are becoming increasingly popular in high-performance variable-frequency drives. In many applications PMSMs are the preferred choice due to their favorable characteristics : high efficiency, compactness, high torque-to-inertia ratio, rapid dynamic response, and simple modeling and control [1], [2]. To achieve proper field orientation in motion control of PMSMs, it is necessary to obtain the actual position of the rotor magnets. Although a position sensor mounted on the motor shaft (encoder, resolver, Hall-effect sensor, etc.) is typically used for this purpose, there is a significant interest in removing those sensors since they are often complex and rather fragile. Their removal thus reduces the overall system cost both in terms of components and maintenance. Moreover the reliability is increased.

Many methods have been presented in the literature for the estimation of the rotor speed and position for the PMSM. The literature is concentrated on three main different approaches [3] :

1) The first approach focuses on estimation of the motor back electromotive force (EMF) and subsequent extraction of position information from this signal [4],[5]. The common characteristic of the back-EMF-based algorithms is that their performance degrades at low speeds, since the back-EMF term vanishes close to standstill.

2) The second approach relies on position dependence on motor inductances due to magnetic saliency. The main techniques are based on the injection of an auxiliary signal to probe the motor electrical subsystem, and use the response to such excitation to estimate the position [6],[7],[8]. The others are not invasive and use directly the main feedback signals which contain position and speed information. Their main advantage is the capability of determining the rotor position at standstill. But they require a significant computing time which degrades their effectiveness at high speed and have low tracking capabilities.

3) The third approach is based on the linear or non-linear state observers, such as Extended Kalman Filter (EKF) or extended Luenberger observer [9],[10],[11],[12]. A suitable design of the observers leads to a resonable level of robustness to parameter variations and measurement noise. Beside, the Kalman filter based algorithm is computationally intensive and the Luenberger observer design is tedious. Nevertheless, the main difficulty is related to the lack of design tools to guarantee the stability of the whole drive.

Generally, the main subject of the papers mentioned above is the determination of gain matrices based on different approaches such as dynamics and/or stability assignment, or insensitivity to parameter uncertainties. But the improvement in the implementation of these observers is seldom studied. The main goal of our paper is to present an efficient implementation of Extended Kalman Filter for position and speed estimation of a PMSM. The linear algorithm developed by C.S. Hsieh and F.C. Chen [13], which is named the optimal two-stage Kalman estimator (OTSKE) is extended to the non-linear estimation case. The two-stage Kalman estimator is composed of two parallel filters : a full order filter and another one for the augmented state. This estimator has the advantage of reducing the computational complexity compared to the classical EKF. The complete equations of this filter are presented and compared to a straight implementation of the classical EKF equations.

This paper is organized as follows. In Section II, the continuous and the discrete model of the PMSM are recalled. In section III, the classical EKF and the non-linear two-stage Kalman estimator equations are detailed. In section IV, the algorithm complexity, experimental and simulation results of both filters are discussed. Finally, a conclusion wraps up the paper.

II. Model of the PMSM

A. Continuous motor model

The salient PMSM is modeled in the standard (dq) reference frame model given as follows :

$$\begin{aligned} \tfrac{d}{dt}X(t) &= A_c(\Theta)\,X(t) + B_c^u(\Theta)\,U(t) + B_c^{\Theta}(\Theta)\,\Theta(t) \\ Y(t) &= C(\Theta)\,X(t) \end{aligned} \tag{1}$$

with

$$
\begin{aligned}
X &= \left[\begin{array}{cc} i_{sd} & i_{sq} \end{array} \right]^t \qquad \Theta = \left[\begin{array}{cc} \omega & \theta \end{array} \right]^t \\
U &= \left[\begin{array}{cc} v_{s\alpha} & v_{s\beta} \end{array} \right]^t \qquad Y = \left[\begin{array}{cc} i_{s\alpha} & i_{s\beta} \end{array} \right]^t \\
A_c(\Theta) &= \left[\begin{array}{cc} -\frac{R_s}{L_d} & \frac{\omega L_q}{L_d} \\ -\frac{\omega L_d}{L_q} & -\frac{R_s}{L_q} \end{array} \right] \\
B_c^u(\Theta) &= \left[\begin{array}{cc} \frac{cos(\theta)}{L_d} & \frac{sin(\theta)}{L_d} \\ -\frac{sin(\theta)}{L_q} & \frac{cos(\theta)}{L_q} \end{array} \right] \quad B_c^\Theta(\Theta) = \left[\begin{array}{cc} 0 & 0 \\ -\frac{\Phi}{L_q} & 0 \end{array} \right] \\
C(\Theta) &= \left[\begin{array}{cc} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{array} \right]
\end{aligned}
$$

In these equations $v_{s\alpha}, v_{s\beta}, i_{s\alpha}, i_{s\beta}$ are the voltages and currents in the (α, β) reference frame, i_{sd}, i_{sq} are currents in the (dq) frame, L_d and L_q are the stator inductances, R_s is the stator winding resistance, and ϕ is the flux produced by the magnets. The angular velocity ω is measured in electrical radians per second (the connection between electrical and mechanical variables is $\omega = P\Omega$, where P is the number of pole pairs) and θ is the electrical position.

B. Discretisation of the motor model

For the digital implementation of an estimator, a discrete-time state space model is required. Provided that the input vector U is nearly constant during a sampling period T_s ($T_s = 200\ \mu s$), the previous continuous model leads to the following discrete-time state space model :

$$
\left\{ \begin{array}{rcl} X[k+1] &=& A(\Theta)\, X[k] + B^u(\Theta)\, U[k] + B^\Theta(\Theta)\, \Theta[k] \\ Y[k] &=& C(\Theta)\, X[k] \end{array} \right.
$$

Tolerating a small discretization error, a first order series expansion of the matrix exponential is used :

$$
\begin{aligned}
e^{A_c T_s} &\approx A = I + A_c T_s \\
A_c^{-1}\, (e^{A_c T_s} - I)\, B_c &\approx B = T_s\, B_c
\end{aligned}
$$

This leads to :

$$
\begin{aligned}
A(\Theta) &= \left[\begin{array}{cc} 1 - \frac{R_s}{L_d} T_s & \frac{\omega L_q}{L_d} T_s \\ -\frac{\omega L_d}{L_q} T_s & 1 - \frac{R_s}{L_q} T_s \end{array} \right] \\
B^u(\Theta) &= \left[\begin{array}{cc} \frac{cos(\theta)}{L_d} T_s & \frac{sin(\theta)}{L_d} T_s \\ -\frac{sin(\theta)}{L_q} T_s & \frac{cos(\theta)}{L_q} T_s \end{array} \right] \\
B^\Theta(\Theta) &= \left[\begin{array}{cc} 0 & 0 \\ -\frac{\Phi}{L_q} T_s & 0 \end{array} \right]
\end{aligned}
$$

This discretised model will be used in the prediction step of the EKF which is designed to estimate the unknown vector Θ defined in the previous paragraph.

III. Extended Kalman filters

A. Classical EKF

The main objective of the estimator is to determine the position and speed of the PMSM. Therefore, those two variables must be concatenated with the previous state vector X. This leads to an augmented observer with a state space vector composed of the currents, the speed and position. This requires to introduce new differential equations to describe the mechanical behaviour. Moreover, to improve the stability of the closed-loop system when the estimates (position and velocity) are used in the controller, it is more judicious to also estimate the load torque T_l. In fact, the knowledge of the load torque is useful to improve the dynamics performances of the drive by a proper torque compensation scheme. Unfortunately, this observer has a heavier computational cost.

Treating $X[k]$ as the full order state and $\Theta[k]$ as the augmented system state, the state space model is described by :

$$
\left\{ \begin{array}{rcl} X^a[k+1] &=& \overline{A}(\Theta[k]) X^a[k] + \overline{B}(\Theta[k])\, U[k] + W[k] \\ Y[k] &=& \overline{C}(\Theta[k]) X^a[k] + B^y(\Theta[k])\, U[k] + \eta[k] \end{array} \right. \tag{2}
$$

with

$$
\begin{aligned}
X^a[k] &= \left[\begin{array}{c} X[k] \\ \Theta[k] \end{array} \right] \qquad \overline{A}(\Theta[k]) = \left[\begin{array}{cc} A(\Theta[k]) & B^\Theta(\Theta[k]) \\ 0 & G(\Theta[k]) \end{array} \right] \\
\overline{B}(\Theta[k]) &= \left[\begin{array}{c} B^u(\Theta[k]) \\ 0 \end{array} \right] \\
\overline{C}(\Theta[k]) &= \left[\begin{array}{cc} C(\Theta[k]) & D(\Theta[k]) \end{array} \right] \qquad W[k] = \left[\begin{array}{c} W^x[k] \\ W^\Theta[k] \end{array} \right]
\end{aligned}
$$

where $G(\Theta[k])$ describes the evolution of the unknown state variable Θ between two time samples.

The process noises $W^x[k]$, $W^\Theta[k]$ and the measurement noise $\eta[k]$ are supposed zero-mean white signal with the following properties :

$$
\left\{ \begin{array}{rcl}
E(W^x[k] W^{x^t}[k-\tau]) &=& Q^x[k]\delta[\tau] \\
E(W^\Theta[k] W^{\Theta^t}[k-\tau]) &=& Q^\Theta[k]\delta[\tau] \\
E(W^x[k] W^{\Theta^t}[k-\tau]) &=& Q^{x\Theta}[k]\delta[\tau] \\
E(\eta[k]\eta^t[k-\tau]) &=& R[k]\delta[\tau] \\
E(W^x[k]\eta^t[k-\tau]) &=& 0 \\
E(W^\Theta[k]\eta^t[k-\tau]) &=& 0
\end{array} \right.
$$

The application of the EKF to the non-linear state space model (2) is described as follows :

$$
\begin{aligned}
X^a[k|k-1] &= \overline{A}[k-1] X^a[k-1|k-1] \\
&+ \overline{B}[k-1] U[k-1] \\
P[k|k-1] &= \overline{F}[k-1] P[k-1|k-1] \overline{F}^t[k-1] \\
&+ Q[k-1] \\
K[k] &= P[k|k-1] \overline{H}^t[k] \\
&+ (\overline{H}[k] P[k|k-1] \overline{H}^t[k] + R[k])^{-1} \\
X^a[k|k] &= X^a[k|k-1] \\
&+ K[k](Y[k] - \overline{H}[k] X^a[k|k-1] - B^y[k] U[k]) \\
P[k|k] &= P[k|k-1] - K[k] \overline{H}[k] P[k|k-1]
\end{aligned}
$$

with

$$\overline{F}[k] = \begin{bmatrix} F(\Theta[k]) & E(\Theta[k]) \\ 0 & G(\Theta[k]) \end{bmatrix} \quad (3)$$

$$\overline{H}[k] = \begin{bmatrix} H_1(\Theta[k]) & H_2(\Theta[k]) \end{bmatrix} \quad (4)$$

$$K[k] = \begin{bmatrix} K^x[k] \\ K^\Theta[k] \end{bmatrix}$$

$$\begin{aligned} F(\Theta[k]) &= \frac{\partial}{\partial X}\Big(A(\Theta[k])\,X[k] + B^\Theta(\Theta[k])\,\Theta[k] \\ &+ B^u(\Theta[k])\,U[k]\Big) = A(\Theta[k]) \\ E(\Theta[k]) &= \frac{\partial}{\partial \Theta}\Big(A(\Theta[k])\,X[k] + B^\Theta(\Theta[k])\,\Theta[k] \\ &+ B^u(\Theta[k])\,U[k]\Big) \end{aligned}$$

$$E(\Theta[k]) = \begin{bmatrix} \frac{i_{sq}L_q}{L_d}T_s & \frac{v_{sq}}{L_d}T_s \\ -\frac{\Phi}{L_q}T_s - \frac{i_{sd}L_d}{L_q}T_s & -\frac{v_{sd}}{L_q}T_s \end{bmatrix}$$

$$G(\Theta[k]) = \begin{bmatrix} 1 & 0 \\ T_s & 1 \end{bmatrix} \qquad D(\Theta[k]) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} H_1(\Theta[k]) &= \frac{\partial}{\partial X}\Big(C(\Theta[k])\,X[k] + D(\Theta[k])\,\Theta[k] \\ &+ B^y(\Theta[k])\,U[k]\Big) = C(\Theta[k]) \\ H_2(\Theta[k]) &= \frac{\partial}{\partial \Theta}\Big(C(\Theta[k])\,X[k] + D(\Theta[k])\,\Theta[k] \\ &+ B^y(\Theta[k])\,U[k]\Big) \end{aligned}$$

$$H_2(\Theta[k]) = \begin{bmatrix} 0 & -sin(\theta)i_{sd} - cos(\theta)i_{sq} \\ 0 & cos(\theta)i_{sd} - sin(\theta)i_{sq} \end{bmatrix}$$

$$P[.] = \begin{bmatrix} P^x[.] & P^{x\Theta}[.] \\ (P^{x\Theta}[.])^t & P^\Theta[.] \end{bmatrix}$$

$$Q[k] = \begin{bmatrix} Q^x[k] & Q^{x\Theta}[k] \\ (Q^{x\Theta}[k])^t & Q^\Theta[k] \end{bmatrix}$$

where, $v_{sd} = cos(\theta)v_{s\alpha} + sin(\theta)v_{s\beta}$, $v_{sq} = -sin(\theta)v_{s\alpha} + cos(\theta)v_{s\beta}$ are voltages in the dq frame.

Table I shows the number of arithmetic operations required at each time sample by the standard EKF algorithm where rough matrix-based implementation is used. The state vector, measurement, input vector and parameter dimensions are respectively n, m, q and p. The total number of arithmetic operations of our algorithm is 650 for a rough implementation. Figures into brackets represent the algorithmic cost of our application.

For a sampling time of $200\mu s$, this would require a CPU clock cycle at least equal to $300ns$. This algorithm's implementation is suitable on powerful processors such as 16 or 32 bits DSP.

B. Non-linear optimal two stage extended Kalman filter

The reduction of the overall cost of the drive is a critical issue in many applications. So as to implement the EKF on less powerful processor for a given sampling period, one solution is to use "*fast algorithm*" such as the optimal two-stage Kalman estimator (OTSKE) proposed by C.S. Hsieh and F.C. Chen [13] in 1999.

TABLE I
KALMAN ESTIMATOR ARITHMETIC OPERATION REQUIREMENT

	Number of multiplications $M(n=4, m=2, q=2)$	Number of additions $A(n=4, m=2, q=2)$
A,B^Θ,B^u and H	function of the system (11)	function of the system (2)
V_{sd} and V_{sq}	(4)	(2)
$X[k\|k-1]$	$n^2 + nq$ (24)	$n^2 - n + nq$ (20)
$P[k\|k-1]$	$2n^3$ (128+2)	$2n^3 - n^2$ (112+1)
$X[k\|k]$	$2nm + mq$ (20))	$2nm + mq$ (20)
$K[k]$	$n^2m + 2nm^2 + m^3$ (72)	$n^2m + 2nm^2 + m^3 - 2nm$ (56)
$P[k\|k]$	$n^3 + n^2m$ (96)	$n^3 + n^2m - n^2$ (80)
Total	(357)	(293)

The initial system of [13] is described by the linear discretized state-space model :

$$\left\{ \begin{aligned} X[k+1] &= A[k]X[k] + B^\Theta[k]\Theta[k] + W^x[k] \\ \Theta[k+1] &= G[k]\Theta[k] + W^\Theta[k] \\ Y[k] &= C[k]X[k] + D[k]\Theta[k] + \eta[k] \end{aligned} \right. \quad (5)$$

where $\Theta[k]$ is a dynamical bias to be estimated. This linear state space model is a general case, where the matrices B^Θ and D give the action of the unknown input Θ that intervenes in the dynamical system or in the measurement input equations. If only the measurement inputs are biased, the matrix B^Θ is zero. Similarly, $D = 0$ when only the dynamics of the system are biased.

The position and speed estimation leads to a non-linear system where the estimated parameters are present in the augmented vector and in the transition matrix $A[k]$. Consequently, this previous model needs to be extended to make it suitable for position and speed estimation of AC machine.

Therefore our model is described by the general non-linear discretized state-space model detailed below where deterministic input vector $U[k]$ is introduced to take into account the voltage source of the machine :

$$\left\{ \begin{aligned} X[k+1] &= A(\Theta[k])X[k] + B^\Theta(\Theta[k])\Theta[k] \\ &+ B^u(\Theta[k])U[k] + W^x[k] \\ \Theta[k+1] &= G[k]\Theta[k] + W^\Theta[k] \\ Y[k] &= C(\Theta[k])X[k] + D(\Theta[k])\Theta[k] \\ &+ B^y(\Theta[k])U[k] + \eta[k] \end{aligned} \right.$$

The two stage EKF is built with two interconnected observers see Fig. 1. The design procedure is allowed because the matrix $\overline{A}[k]$ is an upper triangular type.

As in [13], the non-linear OTSKE is obtained by using a transformation $T[.]$ so that the variance-covariance matrices $\overline{P}[.]$ are diagonal :

$$T(J) = \begin{bmatrix} I & J \\ 0 & I \end{bmatrix} \qquad \overline{P}[.] = \begin{bmatrix} \overline{P}^x[.] & 0 \\ 0 & \overline{P}^\Theta[.] \end{bmatrix} \quad (6)$$

The non-linear OTSKE is obtained by two transformation matrices $T(M[k])$ and $T(N[k])$ so that overlined expressions correspond to vectors and matrices in the new base :

$$\left\{\begin{array}{rcl} X^a[k|k-1] & = & T(M[k])\overline{X}^a[k|k-1] \\ P[k|k-1] & = & T(M[k])\overline{P}[k|k-1]T(M[k])^t \\ X^a[k|k] & = & T(N[k])\overline{X}^a[k|k] \\ K[k] & = & T(N[k])\overline{K}[k] \\ P[k|k] & = & T(N[k])\overline{P}[k|k]T(N[k])^t \end{array}\right. \quad (7)$$

or

$$\left\{\begin{array}{rcl} \overline{X}^a[k|k-1] & = & T(-M[k])X^a[k|k-1] \\ \overline{P}[k|k-1] & = & T(-M[k])P[k|k-1]T(-M[k])^t \\ \overline{X}^a[k|k] & = & T(-N[k])X^a[k|k] \\ \overline{K}[k] & = & T(-N[k])K[k] \\ \overline{P}[k|k] & = & T(-N[k])P[k|k]T(-N[k])^t \end{array}\right. \quad (8)$$

These two matrices $M[k]$ and $N[k]$ are defined respectively by $M[k] = P^{x\Theta}[k|k-1]\,(P^{\Theta}[k|k-1])^{-1}$ and $N[k] = P^{x\Theta}[k|k]\,(P^{\Theta}[k|k])^{-1}$.
Finally, the non-linear OTSKE equations are :

- State and parameter prediction :

$$\begin{array}{rcl} \overline{X}[k|k-1] & = & A[k-1]\overline{X}[k-1|k-1] \\ & + & B^u[k-1]U[k-1]+u[k-1] \\ u[k-1] & = & (A[k-1]N[k-1]+B^{\Theta}[k-1] \\ & - & M[k]G[k-1])\overline{\Theta}[k-1|k-1] \\ \overline{\Theta}[k|k-1] & = & G[k-1]\overline{\Theta}[k-1|k-1] \\ \overline{P}^x[k|k-1] & = & F[k-1]\overline{P}^x[k-1|k-1]F[k-1]^t \\ & + & \overline{Q}^x[k] \\ \overline{Q}^x[k] & = & Q^x[k]-Q^{x\Theta}[k]\overline{M}[k+1]^t \\ & - & M[k+1](Q^{x\Theta}[k]-\overline{M}[k+1]Q^{\Theta}[k])^t \\ M[k] & = & \overline{M}[k] \\ & + & (Q^{x\Theta}-\overline{M}[k]Q^{\Theta})\overline{P}^{\Theta}[k|k-1]^{-1} \\ \overline{M}[k] & = & (F[k-1]N[k-1] \\ & + & E[k-1]_{\Theta[k-1|k-1]})G[k-1]^{-1} \\ E[k] & = & \frac{\partial}{\partial\Theta}(A[k](\overline{X}[k]+N[k]\overline{\Theta}[k]) \\ & + & B^{\Theta}[k]\,\Theta[k]+B^u[k]\,U[k])_{\overline{\Theta}[k|k]} \\ \overline{P}^{\Theta}[k|k-1] & = & G[k-1]\overline{P}^{\Theta}[k-1|k-1]G[k-1]^t \\ & + & Q^{\Theta}[k] \end{array}$$

- State and parameter correction :

$$\begin{array}{rcl} \overline{X}[k|k] & = & \overline{X}[k|k-1]+\overline{K}^x[k](Y[k]-C[k]\overline{X}[k|k-1] \\ & - & (S-\overline{S})\overline{\Theta}[k|k-1]-B^y[k]U[k]) \\ \overline{K}^x[k] & = & \overline{P}^x[k|k-1]H_1[k]^t(H_1[k]\overline{P}^x[k|k-1] \\ & & H_1[k]^t+R)^{-1} \\ \overline{P}^x[k|k] & = & \overline{P}^x[k|k-1]-\overline{K}^x[k]H_1[k]\overline{P}^x[k|k-1] \\ \overline{\Theta}[k|k] & = & \overline{\Theta}[k|k-1] \\ & + & \overline{K}^{\Theta}[k](Y[k]-C[k]\overline{X}[k|k-1] \\ & - & \overline{S}[k]\overline{\Theta}[k|k-1]-B^y[k]U[k]) \\ \overline{K}^{\Theta}[k] & = & \overline{P}^{\Theta}[k|k-1]S[k]^t(H_1[k]\overline{P}^x[k|k-1]H_1[k]^t \\ & + & R[k]+S[k]\overline{P}^{\Theta}[k|k-1]S[k]^t)^{-1} \\ \overline{P}^{\Theta}[k|k] & = & \overline{P}^{\Theta}[k|k-1]-\overline{K}^{\Theta}[k]S[k]\overline{P}^{\Theta}[k|k-1] \\ S[k] & = & H_1[k]M[k]+H_2[k] \\ \overline{S}[k] & = & C[k]M[k]+D[k] \end{array}$$

Now, it is possible to define the original state $\hat{X}$ as the sum of the state $\overline{X}$ with the augmented state $\overline{\Theta}$:

$$\begin{array}{rcl} \hat{X}[k|k-1] & = & \overline{X}[k|k-1]+M[k]\overline{\Theta}[k|k-1] \\ \hat{X}[k|k] & = & \overline{X}[k|k]+N[k]\overline{\Theta}[k|k] \end{array}$$

with $N[k]=M[k]-\overline{K}^x[k]S[k]$.

The initial conditions of this non-linear OTSKE are established with the initial conditions of a classical EKF ($\hat{X}[0|0]$, $\hat{\Theta}[0|0]$, $P^x[0|0]$, $P^{\Theta}[0|0]$, $P^{x\Theta}[0|0]$), so that :

$$\begin{array}{rcl} N[0] & = & P^{x\Theta}[0|0](P^{\Theta}[0|0])^{-1} \\ \overline{X}[0|0] & = & \hat{X}[0|0]-N[0]\overline{\Theta}[0|0] \\ \overline{\Theta}[0|0] & = & \hat{\Theta}[0|0] \\ \overline{P}^x[0|0] & = & P^x[0|0]-N[0]P^{\Theta}[0|0]N^t[0] \\ \overline{P}^{\Theta}[0|0] & = & P^{\Theta}[0|0] \end{array}$$

Table II shows the number of arithmetic operations required at each time sample by the non linear optimal two stage extended Kalman filter algorithm where rough matrix-based implementation is used. The state vector, measurement, input vector and parameter dimensions are respectively n, m, q and p. The total number of arithmetic operations of our algorithm is 514 for a rough implementation, which means a cost reduction of 21% compared to the classical EKF (see Tab. I).

The remaining potential allows parameters tracking capability for the same computational cost or the use of a cheaper or less powerful microcontroller.

IV. Simulation and experimental results

A. Simulation result

A vector control PMSM drive is implemented in the Matlab/Simulink environment. The estimators are first evaluated

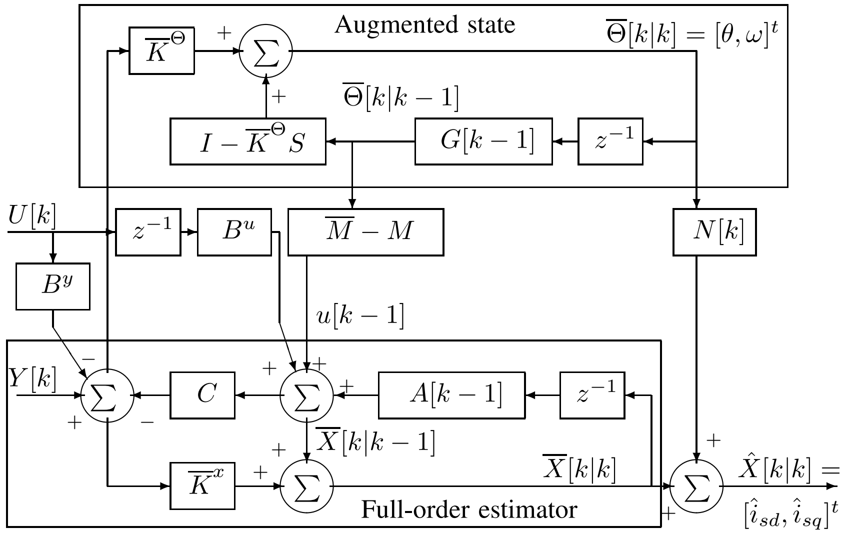

Fig. 1. Block diagram of the two stage extended Kalman estimator

TABLE II
NON-LINEAR OTSKE ARITHMETIC OPERATION REQUIREMENT

	Number of multiplications $M^o(n=2, p=2, m=2, q=2)$	Number of additions $A^o(n=2, p=2, m=2, q=2)$
A,B^{Θ},B^u and H	function of the system (11)	function of the system (2)
V_{sd} and V_{sq}	(4)	(2)
$\overline{X}[k\|k-1]$	n^2+nq (8)	n^2+nq (8)
$u[k-1]$	n^2p+np^2+np (20)	$n^2p+np^2-np+2p^2-p$ (18)
$\overline{P}^x[k\|k-1]$	$2n^3$ (16)	$2n^3-n^2$ (12)
$E[k-1]$	function of the system (2)	function of the system (1)
$\overline{Q}^x[k-1]$	$2n^2p$ (16)	$2n^2p$ (16)
$\overline{M}[k-1]$	$n^2p+np^2+p^3$ (24)	$n^2p+np^2+p^3-np$ (20)
$M[k-1]$	$2np^2+p^3$ (24)	$2np^2+p^3$ (24)
$\overline{\Theta}[k\|k-1]$	p^2 (4)	p^2-p (2)
$\overline{P}^{\Theta}[k\|k-1]$	$2p^3$ (16)	$2p^3-p^2$ (12)
$\overline{X}[k\|k]$	$nm+mp$ (8)	$nm+mp$ (8)
$\overline{K}^x[k\|k]$	$n^2m+2nm^2+m^3$ (32)	$n^2m+2nm^2+m^3-2nm$ (24)
$\overline{P}^x[k\|k]$	n^3+n^2m (16)	$n^3+n^2m-n^2$ (12)
$\overline{\Theta}[k\|k]$	$2mp+nm+mq$ (16)	$2mp+nm+mq$ (16)
$\overline{K}^{\Theta}[k\|k]$	$2p^2m+2pm^2+m^3$ (40)	$2p^2m+3pm^2+m^3-3pm$ (28)
$\overline{P}^{\Theta}[k\|k]$	p^2m (8)	p^2m (8)
$S[k]$	npm (8)	mp (4)
$\overline{S}[k]$	npm (8)	mp (4)
$N[k]$	npm (8)	np (4)
Total	(289)	(225)

in open loop as described in Fig. 2, where estimated speed and position are not included in the vector control strategy. The results are shown in Fig. 3. The main objective of the simulation is to evaluate the equivalence of both estimators. The difference in speed and position estimations between the two estimators is rather null as we can see in Fig. 3. The EKF and the non-linear OTSKE are mathematically equivalent without requiring system constraints [13]. Once can also notice in the upper traces in Fig. 3 the good tracking of the speed and position.

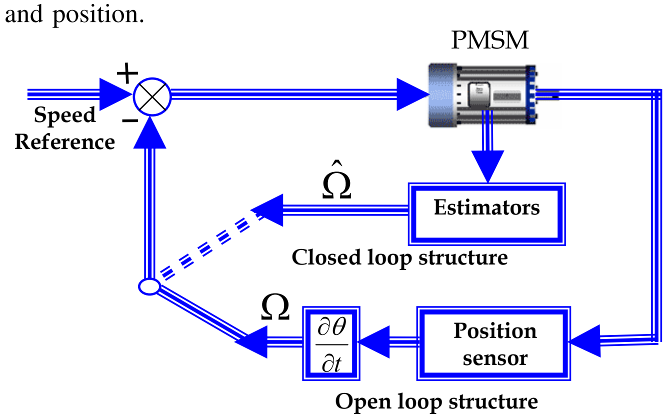

Fig. 2. Open loop and closed loop structure

B. Experimental results

Fig. 4 shows the experimental setup. The PMSM and inverter data are given in Table III. The currents flowing in the stator windings are measured with two Hall Effect current sensors and a 4096 points pulse incremental encoder is used as position sensor. The vector control and the two estimators are implemented on a dSpace® 1104 board using the Matlab-Simulink® software package. The top curves in Fig. 5 show the performances of the speed and position tracking capabilities of the OSTKE estimator compared to the sensor output. The transient performance could be improved by taking into account the friction torque, especially at low speed. The middle curves in Fig. 5 represent the speed and position errors between the sensor and the OSTKE estimator. The differences between the two estimators are represented in the lower curves in Fig. 5. As we can expect the two estimators give the same speed and position estimations and this prove that the two observers are mathematically equivalent

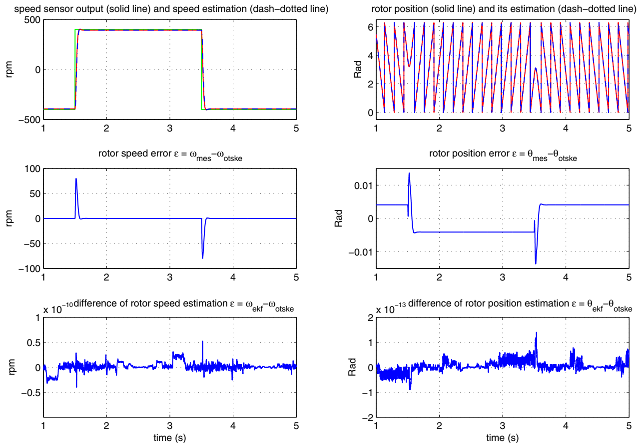

Fig. 3. Simulation result for speed and position estimation

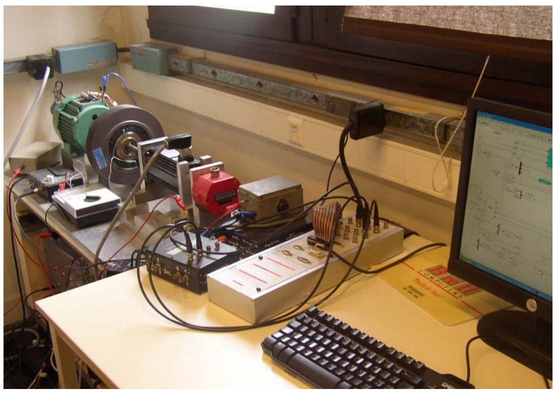

Fig. 4. Experimental setup

V. Conclusion

Sensorless position control of electric drives is an attractive solution particularly in transportation where cost and reliability are key issues. Nevertheless the performances require the estimation of the position and speed. In this paper we have developed a non linear optimal version of the two stage Extended Kalman Filter. Simulation results and laboratory tests on a PMSM drive have demonstrated the feasibility of the approach in open loop while closed loop operation has yet to be performed. Beside the static and dynamic performances, the computational cost is reduced by 21% compared to the classical EKF while keeping the same tracking capabilities. This margin can be used to track more parameters or to implement the algorithm on a cheaper or less powerful target.

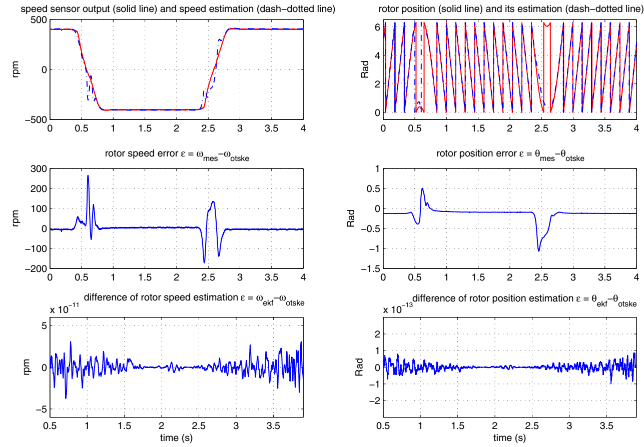

Fig. 5. Experimental result for speed and position estimation

TABLE III
Machine parameters

Rated output power	$P_n = 1,5$kW
Rated torque	$C_n = 3.2$Nm
Rated speed	$N = 3400$tr/min
Rated voltage	$V_n = 260$V
Rated current	$I_n = 5.9$A
Stator resistance	$R_s = 0.255\Omega$
Stator inductances	$L_d = 4, L_q = 3.6$mH
Rated flux	$\Phi_r = 0.17$Wb
Number of pole pairs	$p = 3$
Inertia load	$J = 0,06Kg.m^2$
Viscous coefficient	$f = 0,04$Nm/s

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