Planar Coil Model using Shell Elements Applied to an Eddy-Current Non-Destructive Testing

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***Abstract* — This paper deals with a original kind of planar coil model developed in the finite element method for non destructive testing applications. The model is constructed by the use of degenerated elements (shell elements). An example is implemented with two well known electromagnetic formulations in 2D. The accuracy of the model is established with analytical equations.**

I. INTRODUCTION

Planar printed coils are increasingly used in eddy-current (EC) non-destructive testing (NDT) applications such as detection or material characterization [1-2]. The use of planar printed coils offers many advantages in NDT applications: capability to improve spatial resolution, sensitivity and if it is constructed on a flexible substrate, adaption to complex geometries.

Numerical methods are adapted for the study of electromagnetic problems in EC-NDT applications with complex geometries. Among the numerical methods for the study of planar coils in NDT the finite element method (FEM) is one of the most adapted. However, in the discretization stage, the FEM make a considerable increase in the number of unknowns and, in consequence, in the computation time. These difficulties are related to the meshing process, such as for example, a strong density of elements (increase in the unknown number) or deformed elements in the thin area and its vicinity (ill-conditioned system). In order to eliminate these problems a kind of degenerated elements (shell elements) is used in this paper [3-4].

In the meshing process, the planar coil is considered as a surface, since increase and deformation of elements are avoided. Then, in the stage problem formulation, the degenerated elements are introduced in order to model the planar coil. Finally, a linear equation system is solved and the fields are calculated in the "real" geometry.

II. PROBLEM DESCRIPTION

The basic geometric configuration is a classical problem in EC NDT: a coil placed above a metallic material (Fig. 1). The problem domain Ω is decomposed in three regions: the target material (conducting and/or ferromagnetic) Ω_c, the planar coil domain Ω_0 and the air region. The planar coil domain has a current density j_0 and a thickness denoted by d.

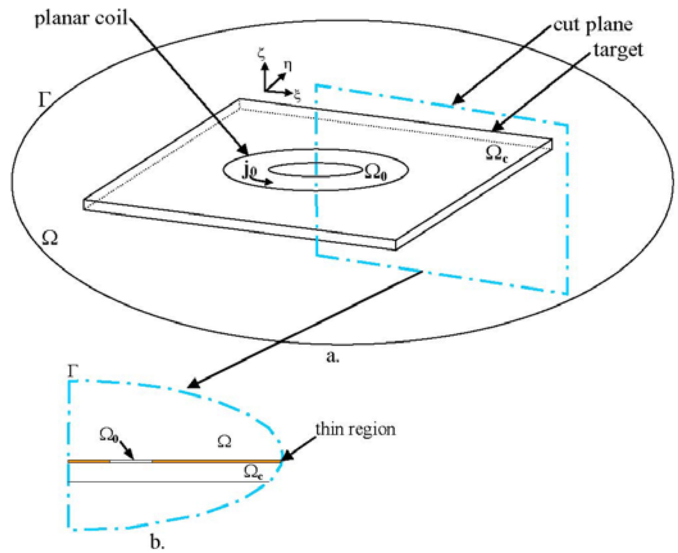


Fig. 1. a.Problem description. b.2D cut plane

The planar coil is geometrically characterized by an aspect ratio L between its thickness d and a characteristic geometrical quantity L_g (in this case L_g is the inner radius):

$$L = \frac{d}{L_g} \,. \tag{1}$$

In the proposed example, the target material is conductive. Thus the field produced by the coil induces EC in the target. The thickness of the coil, d, vary between about 1 micron to 10 millimeters.

III. MAGNETODYNAMIC FORMULATIONS

The physical model of the EC problem is performed by the Maxwell reduced equations (without displacement currents: the capacitance effect is not significant). Among the models of EC problems, an approach based on dual formulations has been proposed by Ren and Razek [5]: the electric and the magnetic formulations.

In the electric formulation, the electric field $\boldsymbol{e}$ is associated to a modified magnetic vector potential $\boldsymbol{a}$*, in the form

$$\boldsymbol{a}^* = -\int_t \boldsymbol{e} dt \,. \tag{2}$$

The weak formulation is based on the Ampere's law, for $t > 0$ in the harmonic regime.

In the magnetic formulation, the magnetic field $\boldsymbol{h}$ is decomposed in two potentials, the electric vector potential $\boldsymbol{t}$ and the scalar magnetic potential ϕ, with

$$\boldsymbol{h} = \boldsymbol{t} - \mathbf{grad}\ \phi. \tag{3}$$

The weak form is based on the Faraday's law applied in the conducting regions and completed by div $\boldsymbol{b} = 0$ for all domains.

IV. SHELL ELEMENTS

The shell elements are a space degeneration of the finite elements. They are geometrical objects that have a *D-1* geometrical dimensions, where *D* is the model geometrical dimension (usually *D=3* or *2*). In the first stage of the discretization process (meshing) since the shell elements are 1 dimensional geometrical objects (lines), the thin region is represented by a middle line (Γ_{sh}). Then, the edge and the nodes in Γ_{sh} are projected on two "virtual" lines (Γ_{sh}^{+} and Γ_{sh}^{-}). The physical behavior inside the planar coil is represented by the shape functions in these lines. Additionally, these projections can be repeated many times in order to stratify the thin region. Using these approximations the formulations are transformed in linear systems in which, the unknowns are the vector circulation along the edges and the scalar quantities at the nodes.

In the first stage of the modeling process, a triangular mesh is constructed. This mesh is highly refined to reduce the error originated by triangular elements. After that, the shell elements are inserted in the thin region. In the example the thin region is stratified in four layers in order to obtain a better approximation of the fields. In this way the error is strongly dependent on the shell elements.

V. RESULTS

The spatial distribution of the real part of magnetic field lines is shown in Fig. 2. A good agreement is observed with the physical behavior: the magnetic field lines encircle the currents. The resistance and reactance of the coil are calculated in the dual formulations and compared with Dodd and Deeds model [6]. The results are shown in Fig. 3.

The field approximation which is made by the linear shape functions in the shell elements explain the different levels of

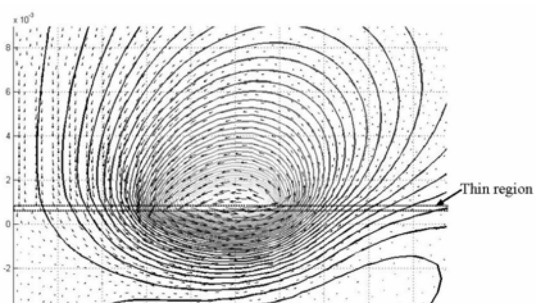
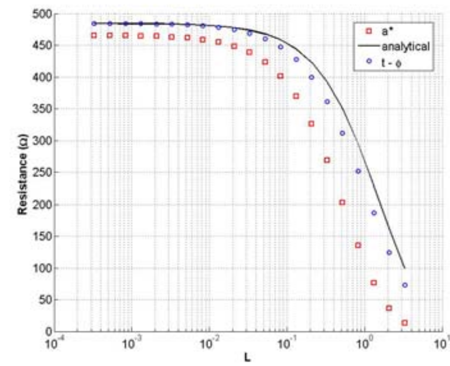
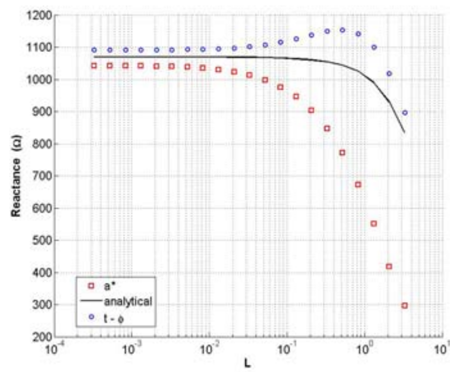


Fig. 2. Spatial distribution of the real part of magnetic flux density vectors and field lines obtained in the ***a**** formulation.

accuracy in the results. The use of the $\boldsymbol{a}^*$ formulation implies a linear variation of the electrical field. The magnetic field $\boldsymbol{h}$, calculated by the curl operator, is then constant and the error is quite important on the reactance (Fig 3b.). In the $\boldsymbol{t} - \phi$ formulation, the behavior of shell elements is analogue: the magnetic field $\boldsymbol{h}$ has a linear variation across the thickness. The variation is more realistic, and the relative error on the



a.



b.

Fig. 3. a. Resistance *vs* aspect ratio (L). b. Reactance *vs* aspect ratio (L).

reactance is less important (Fig 3b.).

The example developed in this digest show that the application of the shell elements in the simulation of planar coils for EC-NDT applications represents an interesting solution when meshing with classical elements is impossible to realize or requires a high number of elements. In the extended paper more details will be provided about the 2D model and a 3D model of the planar coil will be presented.

VI. REFERENCES

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