

Time-domain homogenization of bianisotropic media

Mohamed H. Belyamoun and Saïd Zouhdi

Laboratoire de Génie Electrique de Paris, 11 rue Joliot-Curie
91192 Gif-sur-Yvette, France
Fax: +033--169418318; email: hicham.belyamoun@lgep.supelec.fr

Abstract

In this paper we propose a time-domain homogenization technique for complex media. By using the periodic unfolding method and the finite element method, we simulate the electromagnetic behaviour of a composite bianisotropic material and compute its homogenized electromagnetic parameters. In particular, numerical results are obtained for a Debye-like bianisotropic material.

1. Introduction

Homogenization of a composite spatially periodic material consists in finding the equivalent electromagnetic constitutive parameters that describe the whole composite material regardless of its local variations.

A novel periodic unfolding method of homogenization has been presented in [1] and puts the theoretical basis of the homogenization in the time domain. In particular, this method allows taking into account bianisotropy, chirality, thermal and memory effects. The equivalent electromagnetic parameters of bidimensional structures in time domain have been computed is presented in [2]. And the simulations of [3] expose a static homogenization of three dimensional structures. Our goal was to implement a more general simulation which could homogenize, tridimensional periodic structures, in time-domain.

2. The unfolding homogenization method

Let's consider a bianisotropic material which behaves in accordance to these constitutive laws:

$$\begin{aligned}\vec{D}(t, y) &= \vec{\epsilon}(y)\vec{E}(t, y) + \vec{\xi}(y)\vec{H}(t, y) + \int_0^t \sigma(y)\vec{E}(s, y)ds + \vec{P}(t, y) \\ \vec{B}(t, y) &= \vec{\zeta}(y)\vec{E}(t, y) + \vec{\mu}(y)\vec{H}(t, y)\end{aligned}\quad (1)$$

The description of \vec{P} , the electric macroscopic polarisation is based on a general convolution model, and explicitly depends on the history of the electric field:

$$\vec{P}(t, y) = \int_0^t \nu(t-s, y)\vec{E}(s, y)ds \quad (2)$$

In this study, we choose the Debye model for dipolar polarization [4]. The macroscopic polarisation \vec{P} and the relaxation time τ are linked to electric field after through the following differential equation:

$$\tau \frac{\partial \vec{P}}{\partial t} + \vec{P} = \epsilon_0(\epsilon_s - \epsilon_\infty)\vec{E} \quad (3)$$

Thus, the susceptibility kernel ν is:

$$v(t, y) = \varepsilon_0 \frac{\varepsilon_s(y) - \varepsilon_\infty(y)}{\tau(y)} e^{-t/\tau(y)} \quad (4)$$

The propagation of electromagnetic waves in these media is governed by Maxwell equations, and the theorems established in [1] not only allows us to study a reference cell Y instead of the whole domain Ω , but also ensure the convergence of the electromagnetic field when the studied domain is unfolded:

$$u^\alpha(x) = u(x) + \nabla_y \bar{u}(x, \frac{x}{\alpha}) + \alpha \bar{\bar{u}}(x, \frac{x}{\alpha}) + \dots \quad (5)$$

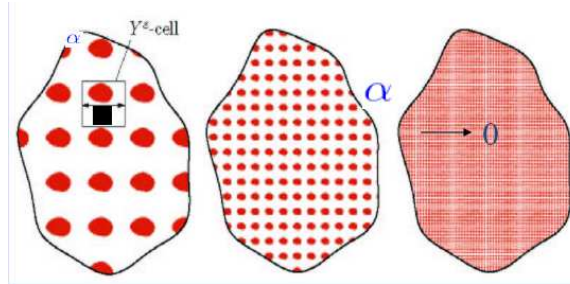


Fig. 1: Unfolding method

The theoretical results of the discretization (of time and space) obtained from [1] and [2] convert the previous cell problem in a linear system, where the unknowns are the corrector subterms w_k^0 , w_k^A and w_k . These subterms are then calculated by the finite elements method.

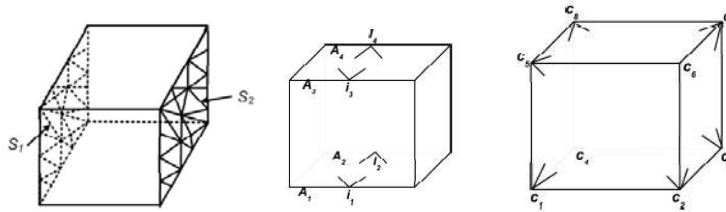


Fig. 2: Associated unknowns

Given the spatial periodicity of the media, several unknowns are identical, thus redundant, in the linear system to be solved. The periodicity is taken into account by eliminating several equations from the system.

These correctors are the key to compute all the effective electromagnetic parameters. For example, the susceptibility kernel is given by an integral on the reference cell:

$$v_{eff}(t_n) = \int_Y \left\{ \begin{aligned} &v(t, y)(e_k + \nabla_y w_k^A(y)) + \\ &\varepsilon_\infty(y) \nabla_y w_k(t, y) + \\ &\int_0^{n\Delta t} (\sigma(y) + v(n\Delta t - s, y)) \nabla_y w_k(s, y) \end{aligned} \right\} dy \quad (6)$$

3. Numerical results

We consider two composite materials, a cube and a cylinder enclosed in a cubic reference cell. The Debye relaxation time is $\tau_i = 31.6ns$ for the inclusions and $\tau_e = 15.8ns$ for the host cell. For both the host medium and inclusions we have $\varepsilon_\infty = 5.5$ and $\varepsilon_s = 78.2$. We computed the homogenized susceptibility kernel of the cubic microstructure and compared it to the susceptibility kernel of the inclusion and the reference cell, an obtained an exponential shape for the homogenized function.

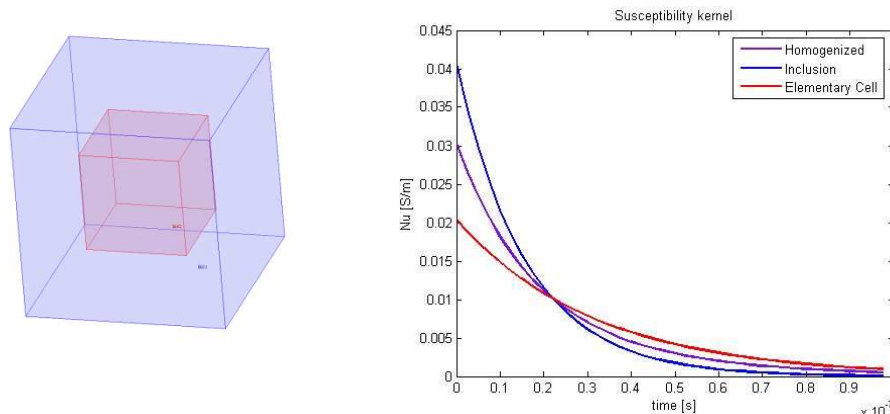


Fig. 3: Homogenized kernel susceptibility in a cubic microstructure

A 2-D circular inclusion with the same relaxation time has been simulated in [2]. We simulated an infinite 3-D cylindrical inclusion and found the same results. We also compared the homogenized susceptibility kernel to the one obtained via this mixing formula [2]:

$$v_{eff}(t) = \left(\frac{f}{v_i(t)} + \frac{1-f}{v_e(t)} \right)$$

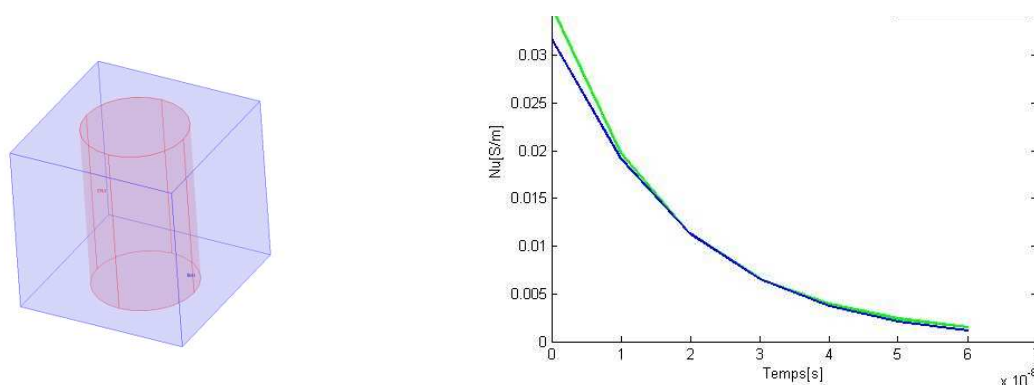


Fig. 4: Homogenized kernel susceptibility in a cylindrical microstructure (in blue) compared to the result of mixing formula (in green)

4. Conclusion

It appears that the unfolding method is a precise way to compute the homogenized parameters for any geometry and a large class of materials. Although it could appear that mixing formulas are sufficient to compute easily the effective parameters, it appears that these analytical methods are designed for non-dense composites with simple elliptic inclusions, and show their limits for complex microstructure shapes.

References

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