Analytical and finite element model for unimorph piezoelectric actuator: Actuator design

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Abstract

The system studied in this paper is a piezoelectric unimorph actuator. Considering the Love-Kirchhoff hypothesis, the linear constitutive relations and the uniaxial stress, we have developed analytic and finite element models for the piezoelectric unimorph actuator in static and dynamic operation. In this analysis we have used the notion of neutral axis for asymmetric structure and Hamilton principle. Furthermore, the structural damping is included in the two models. To evaluate models, results are compared with 3D finite element software. After validation, the actuator design is studied for different passive layers including optimal thickness of the piezoelectric active layer and frequency for the unimorph piezoelectric actuator.

Keywords: unimorph piezo-actuator, analytic and finite elements models, actuator design.

1. Introduction

A piezoelectric unimorph actuator is an asymmetric structure where one piezoelectric active layer is bonded to an elastic passive layer. When the active layer is driven to expand or contract, the passive layer resists against this change, therefore due to the difference in strain between the two layers bending and stretching along the length occurs. The use of piezoelectric ceramic material such as PZT as active layer in an unimorph actuator allows up and down bending with extending and contracting. The change of direction of the applied electric field produces this variation. Compared to electrostrictive unimorph actuator, strain is proportional to the square of the applied electric field, therefore only down bending with extending are produced in the case of electrostrictive unimorph actuator.

Due to the electrical to mechanical conversion, piezoelectric unimorph actuator is used in many applications such as, optical fibre alignment [1], positioning objects, piezoelectric miniature robots [2], deformable mirrors [3], micropumps [4], valves [5] and many others applications. Several papers in the literature are devoted to finite element model and analytical calculation for beam with bonded piezoelectric sensors and actuators. Examples involving the analytical model of piezoelectric unimorph actuator can be found in [2, 6]. Examples using the finite element method for such system have been studied in [7, 8]. Damping coefficient is rarely studied in the literature for analytical models due to the difficulty to resolve the differential equation compared to the finite element one. In this paper, we used Hamilton principle to derivate the analytical and the finite element models, also damping coefficient is added in the two models.

2. Models for piezoelectric unimorph actuator

The equation of motion for the piezoelectric unimorph actuator is derived based on the following assumptions:

1. Euler-Bernoulli hypothesis i.e. uniaxial stress in the x-direction, small deformation and cross section remain perpendicular to the neutral axis after deformation
2. The electric field is assumed to be uniformly distributed in the z-direction {E}= E_3(x,t)

Then, the displacement field becomes

$$\{u\} = \begin{cases} u_1(x,y,z,t) \approx -(z - z_n)\,\partial_x w \\ u_2(x,y,z,t) = 0 \\ u_3(x,y,z,t) \approx w(x,t) \end{cases}$$

Where $w(x,t)$ is the transverse displacement of the neutral axis of the system z_n, it can be calculated by setting to zero the sum of all forces in x-direction over the entire cross-section [9]. Geometric parameters for the system are given in figure 1, here z_n is computed from the bottom of the system: $b\int_0^{t_m} \sigma_1^m(z)dz + b\int_{t_m}^{t_m+t_p} \sigma_1^p(z)dz = 0$

Where σ_1^p and σ_1^m are refered to as stresses induced in the piezoelectric and elastic layers. According to uniaxial stress assumption, Hooke's law under linear-elastic conditions becomes

$\sigma_1^m = c_m \varepsilon_1$, $\sigma_1^p = c_p \varepsilon_1 - e_p E_3 = \frac{1}{s_{11}^E}\varepsilon_1 - \frac{d_{31}}{s_{11}^E} E_3$

Where c_m, s_{11}^E et $d_{31}$are represented in table 1.Utilizing Hook's law, while substituting strain relationship from the last integration, yields: $\int_0^{t_m} c_m (z - z_n)dz + \int_{t_m}^{t_m+t_p} c_p(z - z_n)\, dz = 0$

Upon simplifying, the neutral axis z_n can be obtained as: $z_n = \frac{1}{2}\frac{c_m {t_m}^2 + c_p {t_p}^2 + 2c_p t_p t_m}{c_m t_m + c_p t_p}$

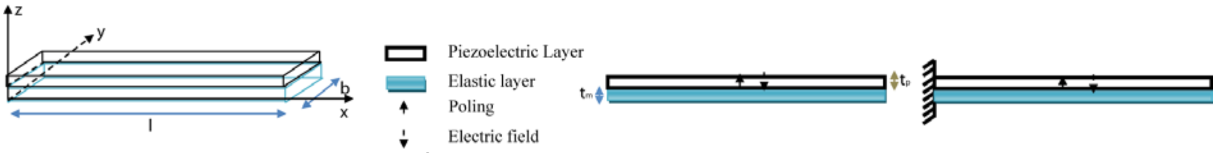

Figure 1: Geometric parameters for the piezoelectric unimorph actuator (3-D and side view) Free at both ends (left side view) and fixed-free (right side view) boundary conditions.

Table 1: Properties for the PZT and Aluminium layers

Materials	Young's modulus (Pa)	Poison's ratio	Volume density (Kg.m^{-3})	Relative permittivity	Piezoelectric constant (m.V^{-1})	Elastic compliances (Pa^{-1})	Length(l) × width(b) × thickness$(t_p$ & $t_m)$ (mm^3)
PZT	/	/	ρ_p = 7900	ε_{33r} = 1350	d_{31} = - 1.3e^{-10}	S_{11} = 1.3e^{-11} S_{12} = -4.76e^{-12}	180 × 17 × 0.27
Aluminum	c_m = 69 × 10^9	υ_m = 0.33	ρ_m = 2700	/	/	/	180×17×0.5

By applying Hamilton principle, we obtain the variational equation; it represents the actuation and sensing operation for the system [8]. The equation of motion is given by the actuation part and with the assumption that no external forces are applied, can be written as

$\int_0^l (\rho I)_{eq}\partial_x \delta w.\ \partial^2{}_t(\partial_x w) + (\rho A)_{eq}\, \delta w.\ \partial^2{}_t w + (EI)_{eq}\, \partial^2{}_x \delta w.\ \partial^2{}_x w + q e_p E_3(x,t)\ \partial^2{}_x \delta w]dx = 0$

Where $(\rho I)_{eq} = (\rho_p I_p + \rho_m I_m)$, $(\rho A)_{eq} = b(\rho_p t_p + \rho_m t_m)$, $(EI)_{eq} = (I_p\ c_p + I_m\ c_m)$ and

$I_m = b\int_0^{t_m}(z - z_n)^2\, dz = b\,\frac{1}{3}\,[(t_m - z_n)^3 + {z_n}^3]$

$I_p = b\int_{t_m}^{t_p+t_m}(z - z_n)^2\, dz = b\,\frac{1}{3}\,[(t_p + t_m - z_n)^3 - (t_m - z_n)^3]$

$q = b\int_{t_m}^{t_p+t_m}(z - z_n)\, dz = b\,\frac{1}{2}\,[(t_p + t_m - z_n)^2 - (t_m - z_n)^2]$

2.1.Analytical model

In static operation, the equation of motion becomes equivalent to the static beam equation:

$$(EI)_{eq}\frac{\partial^2 w(x)}{\partial x^2} = - q e_p E_3$$

With fixed-free boundary conditions, the transverse displacement of the actuator is given by

$$w(x) = \frac{1}{2}\, k\, x^2 \quad \text{where } k = \frac{-\ q e_p E_3}{(EI)_{eq}}$$

To get the dynamic equation of motion in transverse configuration, we ignore the displacement in the x-direction and it becomes equivalent to the dynamic beam equation:

$$(EI)_{eq}\frac{\partial^4 w(x,t)}{\partial x^4} + (\rho A)_{eq}\frac{\partial^2 w(x,t)}{\partial t^2} + c\,\frac{\partial w(x,t)}{\partial t} = - q e_p \partial^2{}_x E_3(x,t)$$

A damping factor c is inerted into the equation of motion, to match the experimental results and is defined as the coefficient of friction related to the actuator length [9]. With free-free boundary conditions, the transverse displacement of the actuator is given by

$$w(x,t) = \sum_{n=1}^{inf} \frac{-q e_p E_3[\partial_x \emptyset_n(l) - \partial_x \emptyset_n(0)]}{(2\pi f_n)^2 \sqrt{(1-\eta_n{}^2)^2 + (2\zeta_n \eta_n)^2}} \emptyset_n(x) \ \sin(\Omega t - \psi_n)$$

E_3, Ω : Constant magnitude and frequency of the applied electric field.

f_n : Resonance frequency and it is equal to $\frac{(\beta_n l)^2}{2\pi l^2}\sqrt{\frac{(EI)_{eq}}{(\rho A)_{eq}}}$, $(\beta_n l)$ are given in table 2.

$\eta_n = \frac{\Omega}{2\pi f_n}$, $\psi_n = \arctan(\frac{2\zeta_n \eta_n}{1-\eta_n{}^2})$, $\zeta_n = \frac{c}{2\,(\rho A)_{eq}(2\pi f_n)}$

$\emptyset_n(x)$: Mode shape and it is equal to $A_n \varphi_n(x)$

$\varphi_n(x) = \sin(\beta_n x) + \sinh(\beta_n x) - \frac{\sin(\beta_n l) - \sinh(\beta_n l)}{\cos(\beta_n l) - \cosh(\beta_n l)}(\cos(\beta_n x) + \cosh(\beta_n x))$

By applying the normalization rule, we can calculate A_n as follow: $A_n{}^2 = \frac{1}{(\rho A)_{eq} \int_0^l \varphi_n(x)^2 dx}$

Table2: Solution of the characteristic equation for free-free piezo-unimorph actuator

n	1	2	3	4	5	6	7	8	9	10
$\beta_n l$	4.73004	7.8532	10.9956	14.1372	17.2788	20.4204	23.5619	26.7	29.8451	33

2.2.Finite element model

The equation of motion can be rewritten in numerical form as:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K_{mm}]\{U\} = -[K_{mv}]E_3 \sin(\Omega t)$$

Where [M] is the mass matrix, $[K_{mm}]$ is the mechanical stiffness matrix, {U} represents the transverse displacement and its derivative at each node with respect to time, $[K_{mv}]$ is the electromechanical coupling stiffness matrix. A damping matrix [C] is integrated in the system via Rayleigh damping [10]. In static operation, the numerical equation of motion is: $[K_{mm}]\{U\} = -[K_{mv}]E_3$ and to determine resonant frequencies and mode shapes, we solve the following equation: $([K_{mm}] - (2\pi f_n)^2 [M])\{U\} = 0$.

3. Actuator design

Before the design of the actuator, a step to validate models is necessary. Validation can be done by comparison of the displacement in static operation, of the resonant frequency and mode shapes and also of displacement in dynamic operation; for the analytical 1D model, finite element 1D model and 3D COMSOL model(figure 2). In the design, the thickness of the piezo-layer is the only one variable parameter for unimorph piezo-actuator. To achieve its optimal performances, a study of the influence of thickness to the displacement and resonant frequency is done for several elastic materials (figure 3). The optimal thickness remains the same if we work in static and in dynamic operations; therefore the study of the optimal thickness is done only in static case.

4. Conclusion

Analytical and finite element 1D models are developed and compared with a 3D COMSOL model for simple asymmetric system which is the piezo-unimorph actuator. After validation process, the finite element 1D model is used to design the actuator. The main advantage of the developed finite element code is that its simplicity compared to the analytical one, also parameters can be online modified, likewise in term of time it is very faster than the 3D COMSOL model. This 1D finite element model will be taken as support for future work to study an asymmetric thin plate with piezoelectric patches actuators and sensors that is few studied in literature.

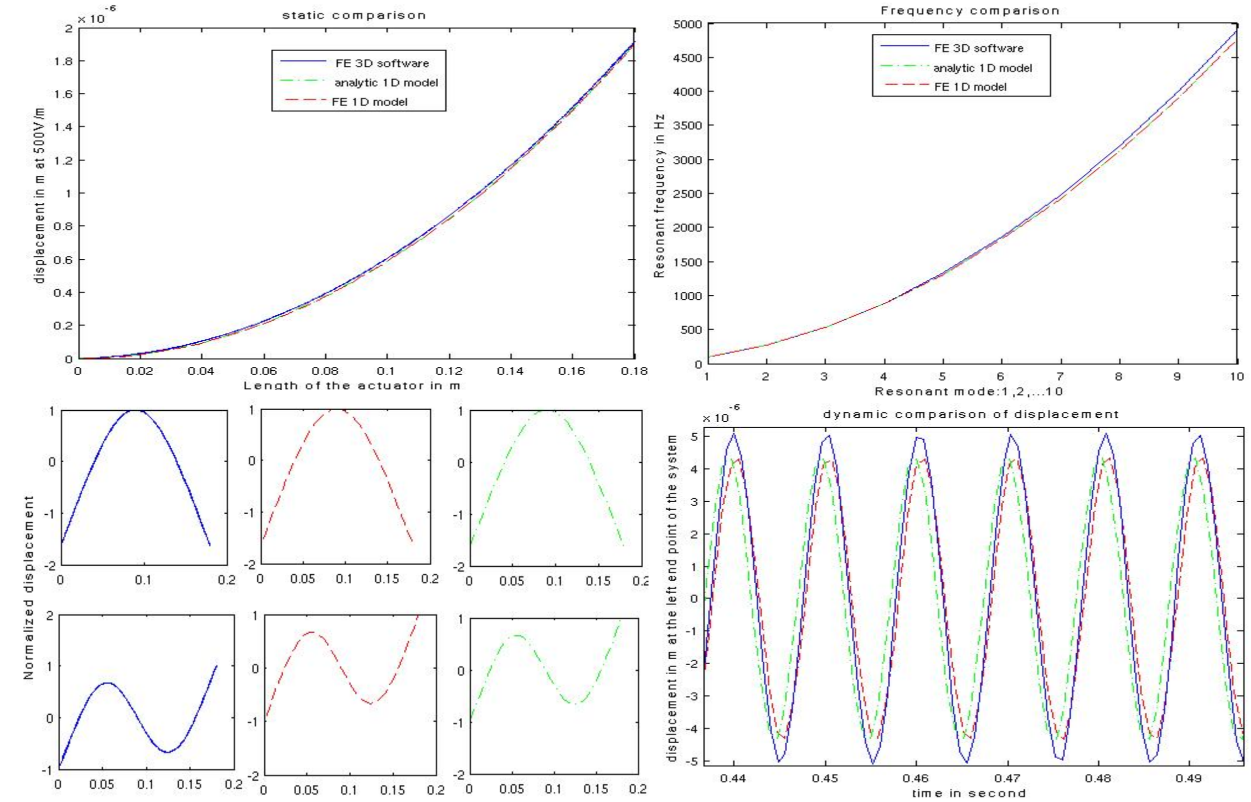

Figure 2: Static comparison for the tip deflection with fixed-free boundary conditions, frequency response for the 10^{th} first mode, first and second mode shapes and dynamic comparison for the displacement, all with free-free boundary conditions

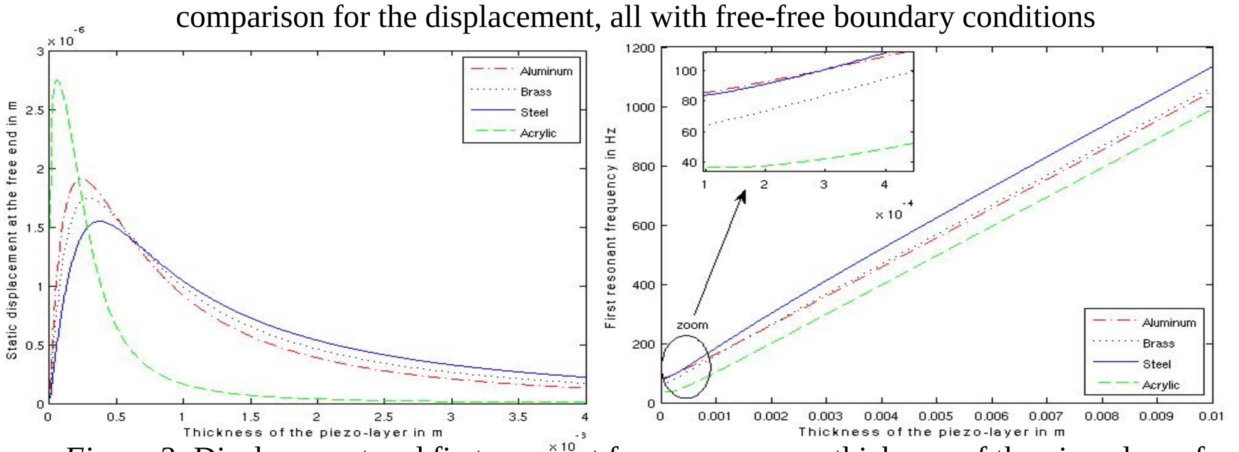

Figure 3: Displacement and first resonant frequency versus thickness of the piezo-layer for several elastic materials (obtained by the FE 1D model)

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