Robust Analysis Towards Robust Optimization in Engineering Design

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This paper deals with robust design in preliminary design. The proposed robust optimization approach is developed for analytical design models coming from the FEM/RSM or from approximation of physical laws. Moreover, it considers variability on design parameters. Towards a global robust and deterministic optimization we propose to implement a robust analysis method to get the links between variability of performances and variability of design parameters. Accuracy and stability of this robust analysis method are tested. Then the robust optimization approach is implemented on an electrical actuator model.

***Index Terms*—Robustness, Design optimization, Statistical analysis, Analytical model.**

I. Introduction

This paper attempts to consider robust design problems. In real engineering, product design problems may be subject to various unavoidable uncertainties appearing everywhere. Uncertainties mainly influence product performances and can lead to wrong products. Uncertainties may affect every design parameter, such as new environmental conditions, geometrical parameters, material properties and so on.

Robust design aims at optimizing product performances in making them less sensitive to parameter variability. This paper deals only with design parameter variations, including material properties. Recently, developments have been made for robust optimization. But they mainly concern stochastic optimization such as GA [1], [2]. We present in this paper how to fill in robust design objectives in a way that fits with deterministic optimization for speeding up the process. Furthermore, in this work we consider that robust design will be reached in the preliminary design stage dealing with analytical models. They come from FEM/RSM [3] or from approximation of physical laws.

The total time for finding a solution using optimization can be approximated by the product of the number of model evaluations and the time required for one evaluation. To decrease this total time, we focus on the evaluation time of the model which integrates uncertainties.

II. The Proposed Approach for Robust Optimization

The approach exposed and implemented in this paper is illustrated on Fig. 1.

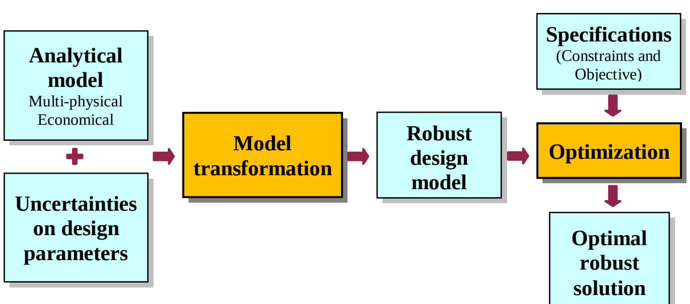

Fig. 1. Proposed approach for robust optimization

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First, the initial model is transformed into a "Robust design model" which integrates uncertainties. Then the "Robust design model" is implemented in an optimization algorithm with specifications for models that integrate uncertainties. Finally, the optimal robust solution is given, providing that it exists.

A. Model transformation

The aim of this transformation is to integrate variability on design parameters in the initial model. Robust analysis methods are introduced to compute the variability of the product performances (output parameters), given the variations of design parameters (model inputs). Several approaches can be used; they involve fuzzy set, interval variables, or random variables. We assume here that design parameters can be represented either by distributions or by nominal values. It is also assumed that random variables are independent.

The most common robust analysis method is the Monte-Carlo simulations [4]. This method can easily be implemented but it requires 10^4 to 10^6 samples to obtain accurate enough results [5]. Hence, the time needed for realizing simulations on a complex model is too high. To reduce time for one model evaluation, Monte Carlo simulations are applied to an approximate model such as the Taylor expansion or the Chaos polynomial [6]. Besides, Latin Hypercube sampling [7] and Importance Sampling [8] techniques aim at reducing the number of evaluations. Another way to characterize product performances variability is to estimate its moments accurately and efficiently and then rebuild the probabilistic distribution function. The k_{th} mathematical moment is expressed by (1):

$$E[\{y(X)\}^k] = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \{y(X)\}^k f_X(X)dX \qquad (1)$$

where y is the output, X is the set of random input parameters and $f_X(X)$ is the joint probability density function of the random inputs. Estimate (1) by numerical integration such as a quadrature rule is practically unfeasible, mainly when the number of input parameters is large. The Univariate dimension-reduction method [9], the Performance moment integration method [10] and the Percentile difference method [10] have been proposed to handle this problem.

All these methods use numerical computations and this leads to the loss of analytical links between input and output moments. But those links are necessary when dealing with deterministic optimization process. The Propagation of Variance method [11] gives analytically the moments expressions. They are obtained from Taylor series expansion around the parameter's mean and for the first and the second order. We have limited this study to the evaluation of the first two moment expressions which are the mean (μ_Y) and the variance (${\sigma_Y}^2$). Those expressions are given for the first order (2) and the second order (3) Taylor expansions.

$$\begin{cases} \mu_Y \approx y(\mu_X) \\ {\sigma_Y}^2 \approx \sum_{i=1}^{n} \left(\frac{\partial y}{\partial X_i}(\mu_X) \right)^2 {\sigma_i}^2 \end{cases} \quad (2)$$

$$\begin{cases} \mu_Y \approx y(\mu_X) + \frac{1}{2} \sum_{i=1}^{n} \frac{\partial^2 y}{\partial {X_i}^2}(\mu_X) {\sigma_i}^2 \\ {\sigma_Y}^2 \approx \sum_{i=1}^{n} \left(\frac{\partial y}{\partial X_i}(\mu_X) \right)^2 {\sigma_i}^2 + \frac{1}{2} \sum_{i=1}^{n} \left(\frac{\partial^2 y}{\partial X_i^2}(\mu_X) \right)^2 {\sigma_i}^4 \\ \quad + \sum_{i<j}^{n} \left(\frac{\partial^2 y}{\partial X_i \partial X_j}(\mu_X) \right)^2 {\sigma_i}^2 {\sigma_j}^2 \end{cases} \quad (3)$$

where y is the model output, X_i are the model inputs, μ_i are the input means, μ_X represents the set of input means and σ_i are the input Standard Deviations.

Equations (2) and (3) are respectively obtained from a linear approximation and a quadratic approximation of the initial model. This means that in engineering design problems where models are rarely linear, equation (3) should be a better approximation of the mean and the variance than equation (2). Moreover, this method requires the evaluation of the Gradient and the Hessian of the model, depending on which Taylor order we choose. Practically, the partial derivatives are often computed by finite differentiation.

Hence, the Mean and the Standard Deviation of each output of the initial model are expressed using the Propagation of Variance method at the second order (Fig. 2).

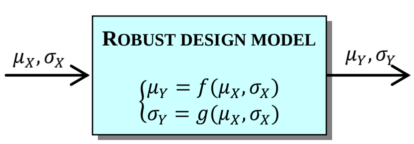

Fig. 2. Robust design model

B. *Optimization*

Once the "Robust design model" is obtained, it can be optimized. This new model has changed and this leads to adapt the specifications (constraints and objective) in a way that integrates variability on parameters. To do so, we propose to base specifications on a quality approach such as the six-sigma strategy [12]:

$$\text{Minimize: } F(\mu_X, \sigma_X, \mu_Y, \sigma_Y)$$
$$\text{Subject to: } g_i\big(\mu_Y(X), \sigma_Y(X)\big) \leq 0$$
$$X_{l_i} \leq \mu_{X_i} \pm 3\sigma_{X_i} \leq X_{u_i}$$

With this formulation, 99,73% of the performance's values are located between the lower and the upper bound. Notice that the designer establishes these new specifications and he requires a specific knowledge of the product that is being designed.

Moreover, the "Robust design model" is a determinist one even if it takes into account stochastic uncertainties. Hence, local and/or global determinist optimization algorithms can be applied on, so as stochastic algorithms.

III. A CASE STUDY

A. *Benchmark: Electromechanical actuator*

Two main experiments have been carried out on a design model benchmark. The studied system is an electromechanical actuator (Fig. 3). It is representative of the type of an actuator design problem in Electrical Engineering. The analytical model has been introduced in [13]. It is characterized by 9 non-linear explicit equations, 21 design continuous parameters, and 12 degrees of freedom (TABLE I).

First, we observed the accuracy and the stability of the Propagation of Variance method. Secondly, we implemented the proposed approach in transforming the model and its specifications in order to optimize it.

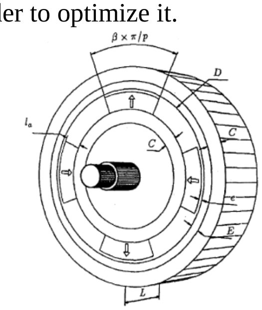

Fig. 3. The electromechanical actuator structure [13]

TABLE I
THE ELECTROMECHANICAL ACTUATOR 'S MODEL

Active parts volume	$V_u = \pi.\frac{D}{\lambda}.(D+E-e-l_a).(2C+E+e+l_a)$
Magnet volume	$V_a = \pi.\beta.l_a.\frac{D}{\lambda}.(D-2e-l_a)$
Losses by Joule effects	$P_j = \pi.\rho_{cu}.\frac{D}{\lambda}.(D+E).E_{ch}$
Form factor	$\lambda = \frac{D}{L}$
Global heating up of the winding	$E_{ch} = k_r.E.J_{cu}^2$
Number of pole pairs	$p = \frac{\pi.D}{\Delta p}$
Leakage coefficient	$K_f = 1.5.p.\beta.\frac{e+E}{D}$
No-load magnetic radial	$B_e = \frac{2.l_a.M}{D.log\left[\frac{D+2.E}{D-2.(l_a+e)}\right]}$
Thickness of the magnetic field	$C = \frac{\pi.\beta.B_e}{4.p.B_{fer}}.D$
Electromagnetic torque	$\Gamma_{em} = \frac{\pi}{2\lambda}.(1-K_f).\sqrt{k_r.\beta.E_{ch}.E}.D^2.(D+E).B_e$

B. *Propagation of Variance method accuracy and stability*

To study the method's accuracy, its results will be compared with those of Monte-Carlo simulations (reference) whereas the method's stability will be obtained by varying the order of magnitude of input Standard Deviation.

Modeling procedure: We first selected input parameters of the electrical actuator model that are likely to vary, such as geometrical parameters or material properties (TABLE II).

Each random input has been arbitrarily chosen as normal distribution for this study. Fixed input values and random input Means are extracted from [13]. We tested the accuracy of these methods for various Standard Deviations, computed from a percentage of the Mean.

TABLE II
DESIGN PARAMETERS OF THE ELECTROMECHANICAL ACTUATOR 'S MODEL, TYPE AND VALUES

Parameter	Type	Value or mean
β: polar arc coefficient	Fixed	0.8
B_{fer}: the magnetic field in iron	Fixed	$1.5\ T$
J_{cu} : current areal density	Fixed	$6.148.10^6\ A/m^2$
k_r: fitting factor	Fixed	0.7
M : magnetic polarization	Fixed	$0.9\ T$
Geometrical variables		
D : bore diameter	Normal random variable	$0.135\ m$
E : winding thickness		$0.0038\ m$
e : mechanical air gap		$0.001\ m$
L : length of iron		$0.07\ m$
l_a : thickness of magnets		$0.0053\ m$
Material properties		
ρ_{cu} : copper resistance	Normal random variable	$17.10^{-9}\ \Omega.m$

Results: All output parameters Means and Standard Deviations are computed using both Monte-Carlo simulations and the Propagation of Variance method. Only the results for output V_u are shown in TABLE III. For each line of the table, the calculation is performed with a different input Standard Deviation (0.5%, 0.83%, 1.67%, 3.33% and 5.55% of the input Mean). Raw results obtained for Monte-Carlo simulations with 10^6 samples are shown in the second column (V_u Mean) and the third column (V_u Standard Deviation). These two columns are considered as the reference for other calculations. The Propagation of Variance method and Monte-Carlo simulations for several sample numbers (10^3, 10^4, 10^5) are realized. Then, we compute the relative error (in %) compared to the Monte-Carlo simulations (10^6) for each method and for each Standard Deviation. Therefore, relative errors are given as a percentage in the last columns of the TABLE III.

TABLE III
RELATIVE ERROR BETWEEN MONTE-CARLO (106), THE PROPAGATION OF VARIANCE METHOD AND OTHER MONTE-CARLO

	Reference: Monte-Carlo 10^6		Propagation of Variance method	
σ_i(%)	$\mu_{V_u}\ (m^3).10^{-4}$	$\sigma_{V_u}(m^3).10^{-}$	μ error (%)	σ error (%)
0,5	5,522	0, 499	0	0,0802
0,83	5,522	0, 8324	0	0
1,67	5,522	1,664	0,0181	0
3,33	5,525	3,331	0	0,0300
5,55	5,529	5,006	0,0181	0,0999

	Monte-Carlo 10^3		Monte-Carlo 10^4		Monte-Carlo 10^5	
σ_i(%)	μ error (%)	σ error (%)	μ error (%)	σ error (%)	μ error (%)	σ error (%)
0,5	0,0181	2,5651	0,0362	0,0400	0	0,1403
0,83	0,0181	5,4181	0,0181	0,8529	0	0
1,67	0,0362	4,6875	0	0,4808	0	0,1803
3,33	0,0724	1,0507	0,0543	0,3903	0	0,0300
5,55	0,3075	0,1798	0,2170	0,2197	0	0,1598

Discussion: Two observations are possible. Firstly, TABLE III shows that the Propagation of Variance method for the Taylor second order gives results close to those obtained using 10^6 simulations based on Monte-Carlo (error max 0.0999%). This observation is valid for Mean and Standard Deviation calculations. Secondly, the Propagation of Variance method is accurate enough, even when input variations become larger. Regarding these remarks, it appears that analytical expressions for μ and σ (3) are highly accurate and have good sensitivity.

C. *Robust optimization*

Modeling procedure: The procedure applied here is the one described on (Fig. 1). Firstly, we have transformed the initial model (TABLE I) into a robust design model which is only composed of Means and Standard Deviations of input and output parameters (Fig. 2). Then we have implemented it in a commercial optimization software called Pro@Design [14]. New specifications were established from initial ones to integrate parameter variability.

All parameters and their type are gathered in the TABLE IV.

TABLE IV
PARAMETER'S TYPE FOR THE OPTIMIZATION PROCESS

Para.	Type	Para.	Type	Para.	Type	Para.	Type
μ_β	Opt.	σ_β	Opt.	μ_p	S	σ_p	S
$\mu_{B_{fer}}$	I	$\sigma_{B_{fer}}$	I	μ_λ	S	σ_λ	S
$\mu_{J_{cu}}$	Opt.	$\sigma_{J_{cu}}$	Opt.	$\mu_{E_{ch}}$	S	$\sigma_{E_{ch}}$	S
μ_{k_r}	I	σ_{k_r}	I	μ_{K_f}	S	σ_{K_f}	S
μ_M	I	σ_M	I	μ_{B_e}	S	σ_{B_e}	S
μ_p	Opt.	σ_p	Opt.	μ_C	S	σ_C	S
μ_D	Opt.	σ_D	Opt.	$\mu_{\Gamma_{em}}$	S	$\sigma_{\Gamma_{em}}$	S
μ_E	Opt.	σ_E	Opt.	μ_{V_u}	S	σ_{V_u}	S
μ_e	Opt.	σ_e	Opt.				
μ_L	Opt.	σ_L	Opt.				
μ_{l_a}	Opt.	σ_{l_a}	Opt.				
$\mu_{\Delta p}$	I	$\sigma_{\Delta p}$	I				

"Para." is for parameter. "Opt." means that the parameter's Mean and Standard Deviation are optimizable. "I" means that the parameter's Mean is fixed to a nominal value while its Standard Deviation is null. Finally, "S" means that the parameter is an output of the model.

Specifications are:

Objective We have decided to minimize $\mu_{V_u} + 3\sigma_{V_u}$ considering that it is the upper bound of V_u's distribution.

Constraints on inputs Each optimizable parameter is constrained as follow:

$$l_{X_i} \le \mu_{X_i} \mp 3\sigma_{X_i} \le u_{X_i} \quad (4)$$

Moreover, the Standard Deviation value is linked to a manufacturing process. Hence, we know the lower value that a parameter's Standard Deviation can take. We constrain the Standard deviation of each optimizable parameter to be higher than a minimum value:

$$\sigma_{min} \le \sigma_{X_i} \quad (5)$$

Constraints on outputs All outputs are constrained as follows except the torque Γ_{em}, the global heating up of the winding E_{ch} and the number of pole pairs p:

$$l_{Y_i} \le \mu_{Y_i} \mp 3\sigma_{Y_i} \le u_{Y_i} \quad (6)$$

As we want a torque under $10N.m$, we have constrained the lower bound of Γ_{em}'s distribution to be higher than $10N.m$:

$$10 \le \mu_{\Gamma_{em}} - 3\sigma_{\Gamma_{em}} \quad (7)$$

We have also constrained the global heating up of the winding E_{ch} to be under $10^{11} A/m$:

$$\mu_{E_{ch}} + 3\sigma_{E_{ch}} \leq u_{E_{ch}} \quad (8)$$

As the number of pole pairs p is a discrete parameter, it was forced to take one of the values that was given.

Results and discussion:

The optimization was realized by using SQP based algorithm and also discrete optimization with some metaheuristics dedicated to Electrical Engineering. 11 iterations were necessary. Results are gathered in TABLE V.

TABLE V
RESULTS OF THE ROBUST OPTIMIZATION

Parameter	Optimal nominal values from the initial model	Mean	Standard Deviation
Optimizable inputs			
D	0,1549	0,1591	0,9883 E-4
β	0,8	0,8030	0,9924 E-3
L	0,06196	0,0637	0,9917 E-4
e	0,001	0,0014	0,9999 E-6
E	0,0033	0,0032	0,9934 E-5
l_a	0,003	0,0030	0,9954 E-5
J_{cu}	0,6619E7	6,637 E6	99,9998
Outputs			
C	0,004931	0,0047	0,1302 E-4
B_e	0,3698	0,3546	0,8107 E-3
p	5	5	0,0031
K_f	0,1606	0,1739	0,4607 E-3
E_{ch}	1,0E11	0,9908 E11	0,3063 E9
λ	2,5	2,5	0,0042
Γ_{em}	10	10,1257	0,0419
V_u	0,5138E-3	0,5407 E-3	0,1467 E-5

Results indicate that the optimization algorithm converges to a solution similar to the one of the initial model. Moreover, this solution is reached by less iteration (17 iterations).

This approach is based on stochastic uncertainties of design parameters and material properties. It should be completed by a worst case analysis to ensure that the resulting solution limits the number of wrong products. However, specifications of the robust optimization are based on a six-sigma theory. Therefore, 99,73% of product must be conform and this leads to a low rate of wrong products.

IV. CONCLUSIONS

According to our study, the Propagation of Variance method is accurate and stable enough. Besides, only two evaluations for obtaining Mean and Standard deviation are required once the model is reformulated. On the contrary, to obtain almost the same accuracy by Monte-Carlo simulations, 10^6 evaluations of the model are required.

Moreover, the robust optimization approach has been applied on implicit and explicit models but we haven't yet studied models including functional (integral function, ordinary differential equation).

REFERENCES

[1] G. Yu-bing, "Robust optimization design of 3D MCM packages for reducing warpage under uncertainty," *Electronic Packaging Technology & High Density Packaging (ICEPT-HDP), 2010 11th International Conference on* , pp.571-575, 16-19 Aug. 2010.

[2] G. Steiner, A. Weber, C. Magele, "Managing uncertainties in electromagnetic design problems with robust optimization," *Magnetics, IEEE Transactions on* , vol.40, no.2, pp. 1094- 1099, March 2004.

[3] X.K. Gao, T.S. Low, Z.J. Liu, S.X. Chen, "Robust design for torque optimization using response surface methodology," *Magnetics, IEEE Transactions on* , vol.38, no.2, pp.1141-1144, 2002.

[4] V. Cavaliere, M. Cioffi, A. Formisano, and R. Martone, « Improvement of MRI magnet design throught sensitivity analysis », *Applied Superconductivity, IEEE Transactions on* , vol.12, no.1, pp. 1413- 1416, 2002.

[5] M. Lemaire, "Structural reliability" *Wiley-ISTE*, 2009.

[6] R. Edwards, A. Marvin, and S. Poter, "Uncertainty analyses in the finite-difference time-domain method", *IEEE Transactions on Electromagnetic Compatibility*, vol. 52, issue 1, pp. 155-163, 2010.

[7] Y. Zhang, H. S. Yoon; C. S. Koh, and D. Xie, "Global optimization of electromagnetic devices combining Latin hypercube sampling experiment and adaptive response surface method," *Electrical Machines and Systems, 2007. ICEMS. International Conference on*, pp.1414-1418, 8-11 Oct. 2007.

[8] R, Kanj, R. Joshi, S. Nassif, "Mixture importance sampling and its application to the analysis of SRAM designs in the presence of rare failure events," *Design Automation Conference, 2006 43rd ACM/IEEE*, pp.69-72, 2006.

[9] S. Rahman and H. Xu, "A univariate dimension-reduction method for multi-dimensional integration in stochastic mechanics", *Probabilistic Engineering Mechanics*, vol. 19, issue 4, pp. 393-408, 2004.

[10] I. Lee, K. K. Choi, L. Du, and D. Gorsich, « Dimension reduction method for reliability-based robust design optimization », *Computers and Structures*, vol. 86, issue 13-14, 2008.

[11] J. Lei, P. Lima-Filho, M. A. Styblinski, and C. Singh, "Propagation of variance using a new approximation in system design of integrated circuits", *NAECON 1998. Proceedings of the IEEE National In Aerospace and Electronics Conference*, pp. 242-246, 1998.

[12] X. Liu, S. Wang, J. Qiu, J. G. Zhu, Y. Guo, Z. W. Lin, "Robust Optimization in HTS Cable Based on Design for Six Sigma," *Magnetics, IEEE Transactions on* , vol.44, no.6, pp.978-981, 2008.

[13] F. Messine, B. Nogarede, and J.-L. Lagouanelle, "Optimal design of electromechanical actuators: a new method based on global optimization", *IEEE Transactions on magnetics*, vol. 34 (2), n°1, pp. 299-308, 1998.

[14] Pro@Design, 2011, «http://designprocessing.free.fr/»

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