The Electromagnetic Actuator Design Problem: An Adapted Interval Global Optimization Algorithm

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This paper presents a deterministic optimization algorithm applied to the optimal design of electromagnetic actuators. The algorithm is based on interval arithmetics and constraint propagation, and aims at solving nonlinear optimization problems by enclosing the global optimum. A new reformulation step is introduced in order to accelerate the convergence of the algorithm and increase solution accuracy. Numerical tests have been performed on the optimal design of electromagnetic actuator.

***Index Terms*—Analytical model, design optimization, nonlinear equation.**

I. Introduction

NOWADAYS, the design optimization topic is generating more and more interest especially with growing environmental issues. In fact, design optimization may lead to substantial savings in material and energy consumption. In the preliminary design of electromagnetic machines, a design model is usually considered. There are two main types of models: exact models from physico-mathematical modeling, and approximated models from RSM, Kriging… [1] A design model is the aggregation of an exact or approximated model and the specifications (see Fig. 1). The aim of the preliminary design phase is to propose a first quantification of the design parameters that will be the basis of the future prototypes. Consequently, proposing the best solution that respects the design constraints (customer, physics, economic…) reduces prototyping costs. F. Messine has proved in [2] the interest of using global optimization methods in the design of electromagnetic actuators; the use of such methods allows for a gain of about 10%.

The interval-based method is one of the most promising deterministic global optimization methods. In fact, the Interval Branch and Bound Algorithm (IBBA) has the property to exactly enclose, with a fixed precision specified by the user, the global optimum and all the corresponding optimizers. Nevertheless, the IBBA, like all deterministic algorithms, may be time-consuming. In addition, as it is based on interval arithmetics, the operators must be adapted to this algebra.

The aim of this paper is to propose a new reformulation adapted to the IBBA and to the preliminary design context that allows to reduce the number of iterations. The algorithm and the reformulation will be tested on the optimal design of electromagnetic actuator.

II. Interval Analysis and Contractors

Interval analysis has been introduced by R. E. Moore in 1966 [3] as an approach to overcome bounding errors. All the usual arithmetic operators have been extended to the interval analysis.

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Before introducing some interval operators, two essential concepts should be defined:

Definition 1: (Interval) An interval is a connected ([1, 2]U[3, 5] is not an interval), closed ([1, 2[is not an interval) subset of $\mathbb{R}$. The set of all intervals of $\mathbb{R}$ will be denoted $\mathbb{IR}$.

Definition 2: (Box) A box is the Cartesian product of n intervals i.e., an interval vector: ([1, 2], [4, 5], [4, 10]) is a tridimensional box. A n-dimension box is an element of $\mathbb{IR}^n$.

Definition 3: (Elementary operators) Let X and Y be two intervals and $\circ \in \{+,-,\times,/\}$:

$$\mathrm{X} \circ \mathrm{Y} = \{\mathrm{x} \circ \mathrm{y} / \mathrm{x} \in \mathrm{X}, \mathrm{y} \in \mathrm{Y}\} \tag{1}$$

Thereby:

$$[a,b] + [c,d] = [a+c, b+d]$$
$$[a,b] - [c,d] = [a-d, b-c]$$

etc…

The other elementary operations have been adapted $(\mathrm{Cos}, \mathrm{Sin}, \log\ldots)$ [4].

We also define the following notations for intervals and boxes. Let i be an interval $[\mathrm{a}, \mathrm{b}]$:

- $left([a,b]) = a$
- $right([a,b]) = b$
- $mid(i) = (a+b)/(2)$ the center of the interval $[a,b]$. For a box, the vector middle of the box intervals. For example, for a box $X = ([1,2][4,5],[4,10]), \quad mid(X) = ((3/2),(9/2),7)$.
- $w(i) = (b-a)$ the width of the interval $[a,b]$. For a box, it is defined by the width of the widest interval. For example, for a box $X = ([1,2][4,5],[4,10]), \quad w(X) = max(1,1,6) = 6$.

A particularly important operator is bisection. In fact, it is the operator that permits to explore smaller parts of the search domain. This operator aims at splitting an interval i (respectively a box B through a specific direction) in order to generate two intervals i_1 and i_2 verifying $i = i_1 \cup i_2$ (respectively two boxes B_1 and B_2 such that $B = B_1 \cup B_2$). The bisection of an interval $[\mathrm{a}, \mathrm{b}]$ generates the two equal intervals $[\mathrm{a}, (\mathrm{a}+\mathrm{b})/(2)]$ and $[(\mathrm{a}+\mathrm{b})/(2), \mathrm{b}]$ [5].

The operators that deal with the constraints are called contractors. The contractors allow to reduce the search space for optimization problems. They are issued from the constraint satisfaction problem solving (CSP) and adapt its constraint propagation techniques. The aim of the contractors is to make pro-

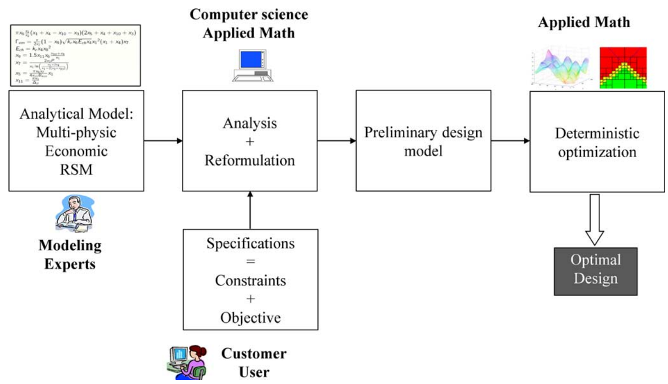

Fig. 1. Preliminary design approach: reformulation and optimization.

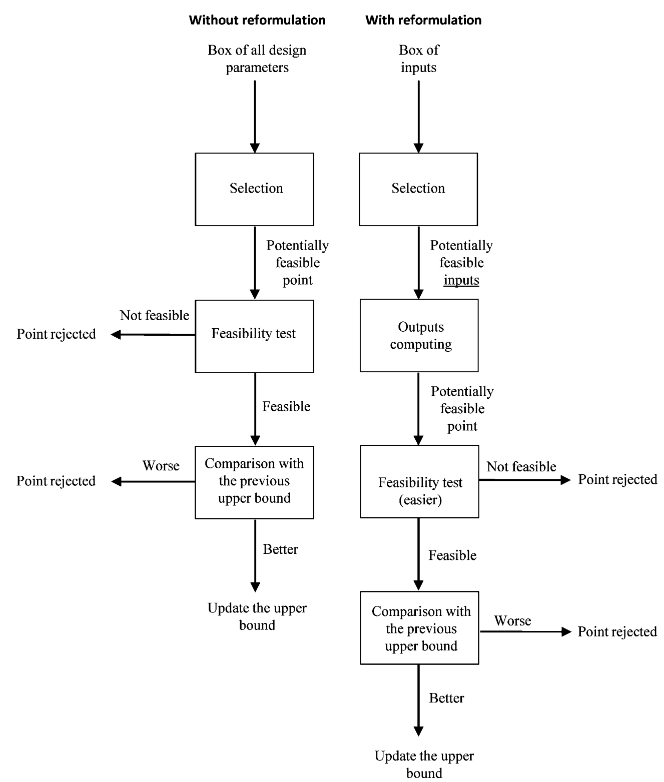

Fig. 2. Upper bound updating step with and without the reformulation.

jections of the constraints on the domains of the variables in order to eliminate the non-feasible parts. Consider the atomic constraint $C: x_1 = x_2 + x_3$ and the intervals D_1, D_2 and D_3 respectively for x_1, x_2 and x_3, a possible contractor can be computed as follows:

$$\begin{cases} D_1 = D_1 \cap (D_2 + D_3) \\ D_2 = D_2 \cap (D_1 - D_3) \\ D_3 = D_3 \cap (D_1 - D_2) \end{cases} \quad (2)$$

Of course, this contractor was possible assuming that the constraint has a simple form. For more complex constraints, two solutions exist:

1. A method based on linearization using centered forms such as Taylor's first order extension to linearize the constraints [6] and make the inversion possible.
2. A method based on the calculus tree commonly named Hull consistency. The central idea is to create intermediate constraints in order to transform a complex constraint into a set of elementary constraints (atomic constraints) [7]. This technique has been tested and has shown good performances [5].

For more details about interval analysis and contractors see [6].

III. Interval Branch and Bound Algorithm and Reformulation

The Interval Branch and Bound Algorithm (IBBA) works by bisecting the search space into small boxes, and discarding the boxes that cannot contain the global minimum. This algorithm is assisted by contractors; that is to say operators that enable to propagate the constraints in order to reduce the search space by eliminating the non-consistent regions.

During the execution of the classic IBBA, the algorithm maintains two lists of boxes; the *WorkList* and the *ResultList*. The *WorkList* contains the boxes that have not been discarded yet and that must be treated, and the *ResultList* contains the boxes that have not been discarded, have a maximum width $\varepsilon_{\text{width}}$ and in which the lower bound of f (the objective function) is higher than the current upper bound plus an $\varepsilon_{\text{bound}}$ precision. At the end of the algorithm, the current upper bound is considered as the global minimum with an $\varepsilon_{\text{bound}}$ precision The boxes of the *ResultList* necessarily contain the global minimum if it exists. For more details about the classic algorithm, see [6], [8].

An important test that permits to discard boxes is the upper bound test (exclusion operator). Consequently, updating the upper bound more often makes the algorithm converge faster. In the classical algorithm, the upper bound is updated with the center of the current box. In fact, the design parameters are selected independently of the constraints, and independently of each other. We notice that for problems with tight constraints, especially equality ones, the updating step is very difficult. In fact, the upper bound must respect all the constraints simultaneously. Consequently, design engineers relax the equality constraints in order to make the feasibility test possible.

The reformulation aims at making the updating step easier, even without relaxing the constraints. The reformulation introduced in this paper takes advantage of specific properties of the analytical models. Currently, we suppose that the constraints of the analytical model are explicit. Knowing that, the constraints have the following form:

$$x_k = g_k(x) \quad (3)$$

where x_k is a design parameter and x the design parameter vector. The whole optimization problem may be expressed as follows:

$$\begin{cases} \min_{x \in X \subseteq \mathbb{R}^n} f(x) \\ x_k = g_k(x), k = 1, \ldots, \mathrm{r} \end{cases} \quad (4)$$

where $X = \{[l_i, u_i], i = 1, \ldots, n\}$ are the bound constraints, $G = \{g_k, k = 1, \ldots, r\}$ is the set of r analytic constraints and $x = (x_1, \ldots, x_n)$ is a realization of all the design parameters. It must be noted that the inequality constraints may be reformulated into equality ones by adding new variables. The design parameters may be classified in two categories: inputs and outputs (see Fig. 2). Following model (4), the output variables are $x_1, \ldots, x_r$. The inputs are the remaining variables $x_{r+1}, \ldots, x_n$. Indeed, for a given input set, the output variables can be computed using analytic constraints. In order to make

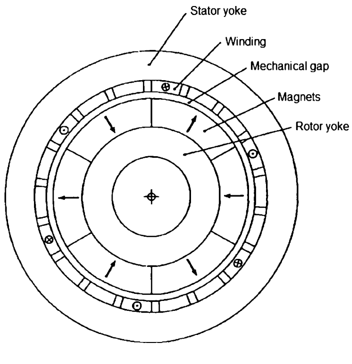

Fig. 3. Electromagnetic actuator structure.

TABLE I
VARIABLE DESIGN PARAMETERS: SIGNIFICATION AND INTERVALS

Symbol	Signification	Interval
$D(m)$	bore diameter	[0.001,0.5]
$B_e(T)$	magnetic field in the air gap	[0.1,1]
K_f	semi-empiric magnetic leakage coefficient	[0.01,0.3]
$J_{cu}(A/m^2)$	current areal density	$[10^5,10^7]$
$e(m)$	mechanical air gap thickness	[0.001,0.005]
$l_a(m)$	permanent magnet thickness	[0.001,0.05]
$E(m)$	winding thickness	[0.001,0.05]
$C(m)$	yoke thickness	[0.001,0.05]
β	polar arc factor	[0.8,1]
λ	length	[1,0.5]

TABLE II
FIXED DESIGN PARAMETERS: SIGNIFICATION AND VALUES

Symbol	Signification	Value
k_r	coil winding filling factor	0.7
$B_{fer}(T)$	magnetic field in the iron	1.5
$E_{ch}(A/m)$	machine warm-up	10^{11}
$\Gamma_{em}(N.m)$	electromagnetic torque	10
$M(T)$	magnetic polarization	0.9
$\Delta_p(m)$	polar step	0.1

this possible, it is necessary to address the equations in a specific order.

This property will be used in the classical IBBA, in order to improve the updating step. As the outputs can be computed via the inputs using the constraints, the IBBA will only seek the optimal inputs. Consequently, the reformulation reduces the problem's dimension. In the updating step, only inputs will be selected independently, the outputs will be then computed. The feasibility test will be replaced by a belonging test. In fact, knowing that, by construction, the combination of the inputs and outputs respects the analytical constraints, the only required condition in order to have a feasible point is to ensure that the outputs belong to their bounds. In the adapted algorithm (see Algorithm I), we no longer consider the whole search space, but only the input box.

Algorithm I Constrained global optimization algorithm with reformulation

1: *Initialize* $X_0^I \leftarrow$ *the initial inputs box*

2: *Initialize* $\tilde{f} \leftarrow +\infty$

3: *Initialize the list* $WorkList \leftarrow (+\infty X_0^I)$*: The elements of the* $WorkList$ *have the form: (lower bound of* f*, Box).*

4: *Initialize the list* $ResultList$: $empty$

while ($WorkList$ *not empty)*

5: *take an element* (v_i, X_i^I) *from* $WorkList$

6: *bisect* X_i^I *into two boxes:* $X_{i1}^I and X_{i2}^I$

for j *from* 1 *to* 2

7: *contract the box using contractors*

8: *compute the lower bound of* f *in* X_{ij}^I : b_{inf}

if $(b_{inf} > \tilde{f})$

9: *discard* X_{ij}^I

else

10: $C_{ij}^I \leftarrow Center(X_{ij}^I)$

11: *compute the corresponding outputs:* C_{ij}^O

if $(f(C) < \tilde{f}$ *and* $X_{ij}^O \in X_0^O)$

12: $\tilde{f} \leftarrow f(C)$

end if

if $(w(X_{ij}^I) < \varepsilon_{\mathrm{width}}$ *and* $\tilde{f} - b_{inf} < \varepsilon_{\mathrm{bound}})$

13: $ResultList \leftarrow ResultList + (b_{inf}, X_{ij}^I)$

else

14: $WorkList \leftarrow WorkList + (b_{inf}, X_{ij}^I)$

end if

end if

end for

end while

IV. NUMERICAL RESULTS

The reformulation has been tested on the optimal design of an electromagnetic actuator (see Fig. 3). The structure of the actuator and its behavior are modeled by the electromagnetic conversion and the flux conservation. The significations of the design parameters are detailed in Tables I and II. The electromagnetic torque is given by the following relation:

$$\Gamma_{\mathrm{em}} = \frac{\pi}{2\lambda}(1-\mathrm{K_f})\sqrt{\mathrm{k_r}\beta\mathrm{E_{ch}}\mathrm{E}\mathrm{D}^2(\mathrm{D}+\mathrm{E})\mathrm{B_e}} \quad (5)$$

$\mathrm{E_{ch}}$ represents the machine heating, and is defined by the relation:

$$\mathrm{E_{ch}} = \mathrm{k_r}\mathrm{E}\mathrm{J}_{\mathrm{cu}}^2 \quad (6)$$

An empirical relation between magnetic leakage coefficient and the geometric dimensions has been established (p stands for the number of pole pairs):

$$\mathrm{K_f} \cong 1.5\mathrm{p}\beta\frac{\mathrm{e}+\mathrm{E}}{\mathrm{D}} \quad (7)$$

The magnetic field in the air gap, supposed exclusively radial, is given by:

$$\mathrm{B_e} = \frac{2\mathrm{l_a M}}{\mathrm{D}\log\left[\frac{\mathrm{D+2E}}{\mathrm{D-2(l_a+e)}}\right]} \tag{8}$$

We give here the expression of the yoke thickness, in which we assume that inter-polar leakage is negligible:

$$\mathrm{C} = \frac{\pi\beta \mathrm{B_e}}{4\mathrm{pB_{fer}}}\mathrm{D} \tag{9}$$

The number of pole pairs is obtained by:

$$\mathrm{p} = \frac{\pi \mathrm{D}}{\Delta_\mathrm{p}} \tag{10}$$

Other constraints related to the variation domains of the design parameters are given; between a small value and a large value on the scale of the considered parameter. These domains are given in Table I.

In the model considered, the optimization consists in minimizing the volume of active parts:

$$\mathrm{V_u} = \pi\frac{\mathrm{D}}{\lambda}(\mathrm{D+E-e-l_a})(2\mathrm{C+E+e+l_a}) \tag{11}$$

The constraints of the electromagnetic actuator model are explicit. Initially, without the reformulation, the output parameters are $\Gamma_{\mathrm{em}}, \mathrm{E_{ch}}, \mathrm{K_f}, \mathrm{B_e}$, C and p. In order to ensure that the outputs vary in intervals so as to make the feasibility test easier, we have rearranged the concerned constraints.

The reformulated design model has the following form:

$$\begin{aligned} \min \quad & V_u = \pi\frac{D}{\lambda}(D+E-e-l_a)(2C+E+e+l_a) \\ \text{s.t.} \quad & \begin{cases} D = \frac{p\Delta_p}{\pi} \\ J_{cu} = \sqrt{\frac{E_{ch}}{k_r E}} \\ B_e = \frac{2l_a M}{D\log\left[\frac{D+2E}{D-2(l_a+e)}\right]} \\ K_f = 1.5p\beta\frac{e+E}{D} \\ \lambda = \frac{\pi}{2\Gamma_{em}}(1-K_f)\sqrt{k_r\beta E_{ch}E}D^2(D+E)B_e \\ C = \frac{\pi\beta B_e}{4pB_{\mathrm{fer}}}D \end{cases} \end{aligned} \tag{12}$$

We compared the numerical results given by the IBBA with and without the reformulation. Table III shows that without the reformulation, we relaxed the equality constraints by $\epsilon_t = 10^{-3}$($\mathrm{p_i} = \mathrm{f(P)}$ is replaced by $\mathrm{p_i} \in [\mathrm{f(P)} - \epsilon_\mathrm{t} \ldots \mathrm{f(P)} + \epsilon_\mathrm{t}]$).

TABLE III
NUMERICAL RESULTS: ACTUATOR OPTIMIZATION

	Without reformulation	With reformulation
Optimum (10^{-4})	6.0743	6.0736
Relaxation	10^{-3}	0
Iterations	60 151 649	470 960
CPU time (s)	1123	13
Updates	12	34
Feasible points	124 434	705 461
	Optimal solution	
$D(m)$	0.12732	0.12732
$B_e(T)$	0.46931	0.45867
K_f	0.18506	0.18090
$J_{cu}(A/m^2)$	6059050.58	6132536.56
$e(m)$	0.001004	0.001
$l_a(m)$	0.00559	0.00521
$E(m)$	0.00389	0.00379
$C(m)$	0.00629	0.00611
β	0.80564	0.8
λ	1.89355	1.82963

We also noted an increase in the number of updates (from 12 to 34), explained by the increase of the number of feasible solutions found (from 124 434 solutions in 60 151 649 iterations to 705 461 solutions in 470 960 iterations). This allows discarding more boxes to be discarded by the bound test, and thus making the convergence faster.

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