Influence of speed variation of a transverse magnetic field on a magnetization of HTS cylinder

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Abstract — In this paper we study numerically the influence of speed variation of a magnetic source on the distribution of current density, magnetization and dissipated energy of a high temperature superconducting cylinder described by a \( J^n \) power law. The results presented come from the resolution of a non linear diffusion problem of electric field by a mixed Finite Element - Finite Volume (FE-FV) discretization method. This method is robust, stable and converges for large values of \( n \). The calculations carried out for \( n \) varying from 1 to 200, show that when the external magnetic field varies quickly from 0 to its maximal value, the maximum values of penetration, the magnetization and the energy dissipation are obtained when the switching of magnetic field occurs. For a periodic magnetic field, we note that any change of the period results in variation of the magnetization and the dissipated energy.

Keywords: high temperature superconductor (HTS), non linear diffusion, constitutive power law, AC losses, magnetization loops, Finite Element Method (FEM), Finite Volume Method (FVM), mixed FE-FV method.

1. INTRODUCTION

Since the use of a non linear \( E(J) \) characteristic given by

\[
\frac{E}{E_c} = \left| \frac{J}{J_c} \right|^{n-1} \frac{J}{J_c}
\]

where \( J \) is the current density, \( E \) the electric field, \( E_c \) the critical electric field, \( J_c \) the critical current density and \( n \) the power law exponent. \( n=1 \) corresponds to a normal conductor and \( n=\infty \) represents the critical state model suggested by Bean

The finite difference method is a robust numerical method to solve the diffusion equation in superconductors. Indeed, it has been successfully used to investigate the influence of temperature and field dependences of the E-J power law on trapped magnetic field in bulk YBaCuO cylinder [3]. In this work, only step applied axial magnetic field is considered. Furthermore, only regular geometries can generally be studied with finite differences.

A new approach has been proposed by the authors. It consists in using coupled finite elements-finite volumes (FE-FV) to solve the electric field diffusion equation in HTS materials [4]. It has been shown that this method converges for large values of \( n \) (up to 200), for different waveforms and speed variations of applied magnetic field.

In this paper, we present the influence of speed variation of an external transverse magnetic field on the induced current density in an infinitely long superconducting cylinder. The characteristics of the HTS cylindrical sample are given in Table 1.

It’s well known that the magnetization loop \( M(H) \) describes a hysteresis cycle whose surface is proportional to the AC losses [5]. The critical current density \( J_c \) is deduced using Bean’s model for which the speed variation of the applied magnetic field doesn’t play any role. Using the magnetization curve \( M(H) \) of Fig. 1 (obtained using the data of Table 1.), \( J_c \) is deduced from the relationship \( J_c = k.A.M \). The constant \( k \) depends on the geometry; \( k=2/3 \pi \) for a cylinder of radius \( R \) and \( k=1/a \) for a slab of thickness \( a \).

In the case where \( J_c \) depends on the flux density \( B \), it has been also shown [6] that the magnetization loop can be used to determine \( J_c(B) \).

Some experimental works have highlighted the frequency dependence in magnetization of high temperature superconducting materials [7].-10]. Our objectives are to describe the influence of speed variation in the case of different kinds of applied magnetic fields and different \( n \) values. Firstly we present the results of current density distribution, magnetization and dissipated energy, obtained with a magnetic field considered as a Heaviside function \( H(t) \) whose variation is the Dirac distribution \( \delta(t) \). In a second numerical application, we present the case of periodic magnetic fields. We compare the current density distributions, the magnetization and the dissipated energy obtained with magnetic fields of same amplitude and different periods.

In the present study, we only consider a constant \( J_c \) and isothermal conditions. Although this last assumption seems to be strong, the actual work allows quantifying the influence of \( n \) as well as the frequency of the applied field on the current density distribution and on the magnetization loop.

Table 1: Characteristics of the superconducting cylinder
2. CYLINDER SUBJECTED TO A TRANSVERSE MAGNETIC FIELD

A) The nonlinear diffusion problem

The superconducting sample is an infinitely long cylinder along z axis, Fig.2. Its characteristics have been given in Table 1. A 2-dimensional model is adopted to study the current density distribution in the cylinder. The applied magnetic field is in the (xy) plan \( B = (B_x, B_y, 0) \) and the induced electric field has only one component along z. The electric field \( E \) satisfies a scalar diffusion equation (2).

\[
\Delta E = \mu_0 \frac{\partial J}{\partial t}
\]

(2)

The differential system is established with a zero initial condition and a Neumann boundary condition (3) derived from the Faraday’s law.

\[
\nabla \cdot \mathbf{E} = \frac{\partial B}{\partial t} = \left( \frac{\partial B_x}{\partial t}, \frac{\partial B_y}{\partial t}, 0 \right) \cdot \mathbf{n}
\]

(3)

where \( \mathbf{n} = (v_x, v_y, 0) \) is the outward normal on the external surface of cylinder.

We consider the case of an applied transverse magnetic field along the x axis, i.e. \( B = (B_0, 0, 0) \), Fig.2. We adopt the notations in (4) and the differential system (5) is solved by a mixed Finite Element- Finite Volume method [4].

\[
\begin{aligned}
\mathbf{u} &= \frac{\mathbf{E}}{E_c} \\
\beta(u) &= u^{1/n} = \frac{J}{J_c} \\
c &= \frac{\mu_0 J_c}{E_c} \\
\frac{\partial \beta(u)}{\partial t} - \Delta u &= 0 \\
\nabla \cdot \mathbf{u} &= \nabla E_c \frac{1}{c} \frac{\partial B}{\partial t}
\end{aligned}
\]

(4)

\[
\begin{aligned}
\mathbf{u}_0 &= 0
\end{aligned}
\]

(5)

B) Case of a magnetic field step

Let’s consider a step of magnetic field proportional to the Heaviside function. At \( t=0 \), it changes suddenly from 0 to its maximum value \( B_{\text{max}} = 15 \, \text{mT} \). We associate this variation to a Dirac distribution \( \delta(t) \). To achieve the numerical calculations, we use a continuum approximation of \( \delta(t) \) given in (6) with \( \varepsilon = 10^{-8} \, \text{s} \). With this approximation, the magnetic field reaches its maximum after a time \( \varepsilon \). Beyond this period, the variation of the magnetic field is zero.

\[
\delta(t) = \lim_{\varepsilon \to 0} \frac{2}{\varepsilon \sqrt{\pi}} \exp \left( -\frac{t^2}{\varepsilon^2} \right)
\]

(6)

1) Current density distribution

Initially, the current density is equal to zero in the cylinder. When a magnetic field is applied, at \( t = 0 \), it penetrates very quickly inside the cylinder. The maximum penetration is attained when the applied magnetic field is \( B_{\text{max}} \). At \( t=5 \, \text{ms} \), one can see that the penetrated part of the cylinder is identical for \( n=25 \) and \( n=200 \) Fig.3.
2) Magnetization over time

Let's define the magnetization as the magnetic momentum with the following expression:

\[
M = \frac{1}{2\pi R^2} \int_{\Omega} \vec{r} \wedge \vec{J}(\vec{r}, t) \, dD \tag{7}
\]

where \(\vec{r}\) is the space vector position.

Since the penetration is rapid, the magnetization over time \(M(t)\) quickly reaches its maximum value. We note that the magnetization reaches 11A.mm\(^{-1}\) for \(n = 25\), and 7A.mm\(^{-1}\) for \(n = 200\), Fig.4.a. Notice that this value corresponds to the one obtained using Bean model, Fig.1. The decrease observed in \(M(t)\) is a translation of the relaxation phenomenon provoked by dissipation in the material, it is more pronounced for \(n = 25\). Indeed, when \(t>0\) the magnetic field does not vary. For \(n=25\) the maximum of \(M(t)\) is obtained with the ratio \(J/J_c = 1.6\). At \(t=5\text{ms}\) this falls to \(J/J_c = 0.98\), while for \(n=200\) the ratio \(J/J_c\) remains close to 1. This explains the drop of the magnetization obtained at \(n = 25\) below that obtained at \(n = 200\), Fig.4.b.

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![Fig. 3. Current density distributions obtained with a step of magnetic field at \(t=5\text{ms}\): (a) \(n=25\) and (b) \(n=200\)

![Fig. 4: Magnetization over time for \(n=25\) and \(n=200\): (a) for \(t \in [0, 10\text{\mu s}]\) and (b) for \(t \in [0.1\text{ms}, 10\text{ms}]\)

![Fig. 5. Dissipated energy in time interval [0, 10\text{ms}] obtained with a step of magnetic field for \(n=25\) and \(n=200\).]
3) Dissipated energy
The dissipated energy is calculated using the following expression:

\[ W_{\text{dissip}} = \int_0^t \vec{E} \cdot \vec{J} ds \]  

(8)

For the values \( n=25 \) and \( n=200 \), we note that the major part of dissipated energy is obtained during switching of magnetic field from 0 to its maximum value \( B_{\text{max}} \). When \( t>0 \), the magnetic field does not vary and the dissipated energy slightly increases. It is clearly higher for \( n=25 \) than for \( n=200 \), Fig.5.

3. CYLINDER SUBJECT TO A PERIODIC TRANSVERSE MAGNETIC FIELDS

In this section, we study the influence of the period of the applied magnetic field on the penetration of the current density, the magnetization and the dissipated energy. We consider triangular and sinusoidal magnetic fields with the same amplitude \( B_{\text{max}} = 15\text{mT} \) and different time periods (or frequencies): \( T = 2\text{s}, T = 0.02\text{s} \) and \( T = 0.001\text{s} \).

A) Penetration
The current density distribution is plotted at \( t=T/4 \). For \( n=25 \) the results obtained with both kinds of magnetic fields show that the decrease of period \( T \) leads to decreasing of the ratio \( J/Jc \) and increasing of the penetration depth, Fig.6 and Fig.7.

When \( n \) is large, the influence of speed variation of the magnetic field disappears and the current density distributions are identical regardless of the period, Fig.8. These results are consistent with those issued from Bean model for which \( n=\infty \). We find that for \( T=0.02\text{s} \) and \( T=0.001\text{s} \), penetration with \( n=200 \) is larger than for \( n=25 \). On the other hand, for \( T=2\text{s} \), penetration with \( n = 25 \) is larger than for \( n=200 \).

![Fig. 6: Current density distributions obtained for \( n = 25 \), with sinusoidal magnetic fields of different periods: (a) \( f = 0.5\text{Hz} \), (b) \( f=50\text{ Hz} \) and (c) \( f=1\text{kHz} \).](image-url)
Fig. 7: Current density distributions obtained for \( n = 25 \), with triangular magnetic of different period: (a) \( T=2s \), (b) \( T=0.02s \) and (c) \( T=0.001s \).

Fig. 8: Current density distributions obtained for \( n = 200 \), with different magnetic fields of same period: (a) Triangular (b) Sinusoidal

B) Magnetization loops

The applied magnetic field is \( B_{\text{max}} = 50 \) mT, a value for which the cylinder is in a complete penetration state. We compare the magnetization loops \( M(H) \) obtained for three periods: \( T = 2s, T = 0.02s \) and \( T = 0.001s \).

We observe that the variation of magnetization \( \Delta M \) decreases with increasing period. The highest \( \Delta M \) and \( J/Jc \) are obtained with the magnetic field of period \( T=0.001s \), Fig.9.

For the three periods, \( \Delta M \) is different from the value given by the Bean model \( (\Delta M_{\text{Bean}} = 14.15A.mm^{-1}) \), Fig. 1. It is smaller when \( T = 2s \) \( (\Delta M_{T=2s} = 12.9A.mm^{-1}) \) and higher for the two other periods \( (\Delta M_{T=0.02s} = 15.5A.mm^{-1} \) and \( \Delta M_{T=0.001s} = 17.5A.mm^{-1} \)).

Similarly, the amplitudes of applied magnetic field needed to achieve full penetration are different from the calculation with Bean model, \( B_{\text{p,Bean}} = 17mT \). The value obtained for \( T=2s \) is 16.2mT; it is lower than the Bean value. For \( T=0.02s \), the obtained value is 23.5mT, and for \( T = 0.001s \), the value is 26.3mT.
For \( n = 200 \) the variation \( \Delta M \) is the same for all three periods (\( \Delta M = 13.90 \text{A.mm}^{-1} \)). It is very close to the variation obtained by the Bean model. When \( n \) is large, the influence of the period of applied magnetic field on the magnetization is negligible, Fig. 10.

In Figure 11, we compare magnetization over time obtained with the three kinds of magnetic fields. For \( n=25 \), only the magnetization with the triangular magnetic field do not decrease, others grow to their maximum then decrease, Fig.11.a. For \( n=200 \), the influence of speed variation of magnetic field disappears and the three obtained results become identical after a while, Fig.11.b.

![Fig. 9](image1.png)

Fig. 9. Magnetization loops \( M(H) \) obtained with magnetic fields of same amplitude and different periods, for \( n = 25 \): (a) Sinusoidal (b) Triangular

![Fig. 10](image2.png)

Fig.10. Magnetization loops \( M(H) \) obtained with magnetic fields of same amplitude and different periods, for \( n = 200 \): (a) Sinusoidal (b) Triangular

**C) Dissipated energy**

For the studied periodic magnetic fields, we compare the dissipated energy on \( \tau \in [0, 1.25] \).

In the case of \( n=25 \), the dissipated energies on the first half period are equivalent, but beyond this time interval, a gap appears, Fig.12. We note that the magnetic field of period \( T=2s \) have the lowest dissipated energy. It gives the lowest ratio \( J/J_c \) in full penetration.

For \( n = 200 \), the dissipated energies are independent of the period \( T \) and the field waveform, Fig. 13. The numerical results of the dissipated energy on a period are very close to the value given by the Bean model. We obtain 0.0095mJ and the Bean model gives 0.01mJ [5]. When \( n \) is large, there is no effects of the speed variation of magnetic field on dissipation in the material. Again, this is consistent with Bean theory.
Fig. 11. Magnetization over time obtained with different applied magnetic fields, a step, a triangular, and sinusoidal: (a) for \( n = 25 \) and (b) for \( n = 200 \)

Fig. 12: Dissipated energy versus \( \tau = t/T \), obtained for \( n = 25 \) with magnetic fields of equal intensity and different periods: (a) Sinusoidal and (b) Triangular
4. CONCLUSION

This study allowed quantifying the influence of the power law exponent n as well as the applied field period and its waveforms on the current density distribution and the magnetization loops of a superconducting cylinder. The developed numerical technique that uses a mixed FEM-FVM coupling scheme makes it possible to carry on these investigations for n-values up to 200.

It has been shown that for n=25, the induced current density, the magnetization and the dissipated energy depend on the applied magnetic field waveforms (step, triangular and sinusoidal). This value of n is typical for HTS materials.

The case n=200 is typical for LTS material for which the Bean theory applies. The numerical results are consistent with those obtained using the Bean model in determining the magnetization and the dissipated energy.

A potential application of the obtained results is to determine the most suitable field waveform and frequency in the characterization process of superconducting materials.

Another important issue concerns the influence of the magnetic field and the temperature on the studied quantities. These studies are under investigation by the authors.

REFERENCES