Robust Adaptive PI Stabilization of a Quadratic Converter: Experimental Results

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***Abstract*—In a previous paper the authors proved that a large class of switched power converters can be globally asymptotically stabilized with a simple linear proportional-integral (PI) controller around a suitably defined output. An explicit formula to construct the desired output, which is a linear combination of the converter states, is also given. Moreover, if the load resistance is unknown, an adaptation mechanism is added to the PI preserving the stability properties. In this paper this procedure is applied to a quadratic boost converter yielding a very simple control that is easy to tune. Experimental results assessing the performance of the proposed adaptive PI are shown.**

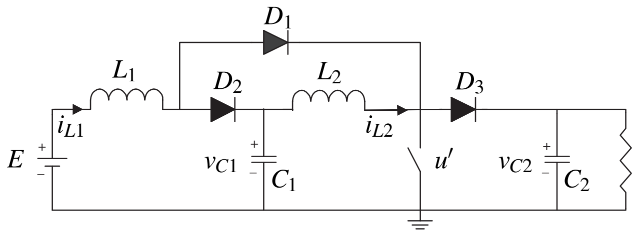

Figure 1: Schematic of the quadratic converter

I. Introduction

The most commonly used approach to regulate the behavior of power converters is with classical linear feedback controllers, in particular, the well–known proportional-integral (PI) control. The controller design is usually based on the linear approximation of the nonlinear dynamics of the converter. Due to the error introduced by this approximation it is clear that the controller has to be de–tuned to ensure a stable behavior in a wide range of operation—yielding an overall below–par performance. Other design methods, such as sliding mode control and one-cycle control, make use of the pulsed and nonlinear nature of switching converters and achieve instantaneous control of the average value of the chopped voltage or current, but the transient performance and noise sensitivity of these design may be unacceptable in some applications.

In [1] the problem of designing simple linear PI controllers with guaranteed stability properties for the full nonlinear model of a large class of switched power converters was studied. The key contribution was the identification of an output, which is a linear combination of the converter states, with the properties that it defines a passive map (with inputs the switch positions), and that convergence to zero of the output implies convergence of the converter state to the desired equilibrium. The result was later extended in [2] where an explicit formula to construct the desired output is given. Moreover, to compensate for the uncertainty in the load resistance, an adaptation mechanism was added to the PI preserving the stability properties.

The objective of this paper is to apply the methodology of [2] to a quadratic converter. This is a novel topology that has attracted the attention of several researchers because it can achieve high amplification rates using only one switch, which makes it more efficient than the standard cascade of boost topologies. It is shown in the paper that the procedure yields a very simple control that is easy to tune. Experimental results comparing the performance of the proposed PI are shown.

The remainder of this paper is organized as follows. Section II discusses the model of the quadratic converter. The adaptive PI is given in Section III. Section IV presents the configuration of the test-bench that was implemented. Experimental validation is given in Section V. Conclusions are drawn in Section VI.

II. Proposed Quadratic Converter

The quadratic converter that we study in the paper is shown in Fig. 1. Its dynamic behavior is described with the averaged

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model given by the equations

$$\begin{aligned} \dot{i}_{L1} &= \frac{1}{L_1}(E - v_{C1}u) \\ \dot{i}_{L2} &= \frac{1}{L_2}(v_{C1} - v_{C2}u) \\ \dot{v}_{C1} &= \frac{1}{C_1}(i_{L1}u - i_{L2}) \\ \dot{v}_{C2} &= \frac{1}{C_2}\left(i_{L2}u - \frac{1}{r_L}v_{C2}\right) \end{aligned} \tag{1}$$

where i_{L1}, i_{L2} are the currents in the inductances, v_{C1}, v_{C2} are the voltages in the capacitors, L_1, L_2, C_1 and C_2 are the values of inductances and capacitances, respectively, r_L is the load, E the input voltage, and $u = 1 - u'$ is a continuous control equal to the slew rate of the pulse width modulator (PWM). To simplify the analysis the inductors are assumed lossless.

For the controller implementation the full state is assumed measurable and the input voltage known. On the other hand, none of the parameters needs to be known and, in particular, an adaptation scheme will be added to estimate the load resistance.

To derive a PI controller for the quadratic converter we will apply the technique described in [2]. Towards this end, the model (1) is expressed in port–Hamiltonian form [4]

$$\dot{x} = (J_0 + J_1 u - R)\frac{\partial H}{\partial x} + B,$$

with the definitions

$$x = \begin{pmatrix} i_{L1} & i_{L2} & v_{C1} & v_{C2} \end{pmatrix}, \; B = \begin{pmatrix} \frac{E}{L_1} & 0 & 0 & 0 \end{pmatrix}^\top$$

$$R = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r_L C_2^2} \end{pmatrix}, \; Q = \begin{pmatrix} L_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & C_1 & 0 \\ 0 & 0 & 0 & C_2 \end{pmatrix}$$

$$J_0 = \frac{1}{C_1 L_2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$J_1 = \begin{pmatrix} 0 & 0 & -\frac{1}{L_1 C_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_2 C_2} \\ \frac{1}{L_1 C_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_2 C_2} & 0 & 0 \end{pmatrix}.$$

where

$$H(x) = \frac{1}{2}x^\top Q x$$

is the energy stored in the circuit.

III. PROPOSED ROBUST PI CONTROL

The goal of the control is to regulate the voltage v_{C2} across the load around a constant value v_d, which is equivalent to regulation of the capacitor voltage x_4 to the constant value $x_4^\star = v_d$. The admissible equilibria of the system (1) can be parameterized by the reference $x_4^\star$ as follows

$$x^\star := \begin{bmatrix} \frac{1}{r_L (u^\star)^2} & \frac{1}{r_L u^\star} & u^\star & 1 \end{bmatrix}^\top x_4^\star$$

where $u^\star = \sqrt{\frac{E}{x_4^\star}}$ is the corresponding constant control.

A. Identifying a suitable output

It is shown in [2] that the output signal

$$\tilde{y} = -\sqrt{E v_d}x_1 - v_d x_2 + \frac{v_d^2}{E r_L}x_3 + \frac{v_d}{r_L}\sqrt{\frac{v_d}{E}}x_4, \tag{2}$$

satisfies the following properties.[1]

P1 The mapping from the incremental input $\tilde{u} = u - u^\star$ to the output $\tilde{y}$ is passive. That is, the system (1), (2), satisfies the power balance inequality

$$\dot{V} \leq \tilde{u}^\top \tilde{y},$$

with

$$V(\tilde{x}) = \frac{1}{2}\tilde{x}^\top Q \tilde{x}$$

the storage function.

P2 The output $\tilde{y}$ is zero–state detectable. That is, the following implication is true

$$\tilde{y}(t) = 0 \quad \Rightarrow \quad \lim_{t\to\infty} x(t) = x^\star.$$

B. Proposed PI control

As discussed in [2] the two aforementioned properties allows us to prove that the equilibrium $x^\star$ can be rendered *globally asymptotically stable* with the PI controller:

$$\begin{aligned} \dot{z} &= \tilde{y} \\ u &= -K_p\tilde{y} - K_i z, \end{aligned}$$

where K_p, K_i are positive tuning gains. It is important to underscore that the only parameters that are required for the implementation of the controller are r_L and E, and that the tuning gains can take arbitrary positive values—hence the "robust" qualifier claimed for the controller.

Even though it is reasonable to assume that the voltage E is known, the load resistance may be highly uncertain. To further robustify the proposed controller an adaptive version of the PI controller is constructed as follows. First, notice that $x^\star$ is linear in the unknown conductance $\theta := \frac{1}{r_L}$. Hence, the passive output $\tilde{y}$ can be written in the linear regression form

$$\tilde{y} = \psi_0(x)\theta + A_0(x), \tag{3}$$

with

$$\begin{aligned} A_0(x) &= -\sqrt{E v_d}x_1 - v_d x_2, \\ \psi_0(x) &= \frac{v_d^2}{E}x_3 + v_d\sqrt{\frac{v_d}{E}}x_4. \end{aligned}$$

In [2] three estimators for θ, with different stability properties, are proposed. All estimators are parameterized by two tuning

[1] See Section IV of [2] for more details.

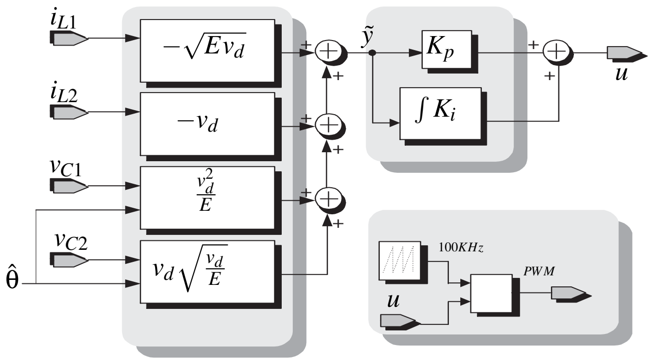


Figure 2: Diagram of the proposed control with unknown load

gains, λ and γ, which can take arbitrary positive values. First, a classical model reference estimator is a second order systems of the form

$$\begin{aligned} \dot{\hat{\chi}} &= -\lambda(\hat{\chi} - x_4) + \frac{1}{C_2}(ux_2 - \hat{\theta}x_4) \\ \dot{\hat{\theta}} &= \gamma x_4(\hat{\chi} - x_4). \end{aligned}$$

Second, an immersion and invariance (I&I) estimator, derived using the techniques studied in the book [6], is given by the first order equation

$$\begin{aligned} \hat{\theta} &= \gamma\xi - \lambda x_4{}^2 \\ \dot{\xi} &= \frac{\lambda}{\gamma C_2}\left(ux_2 - \hat{\theta}x_4\right)x_4 \end{aligned}$$

Finally, a second I&I estimator, which assumes x_4 is bounded away from zero, is described by

$$\begin{aligned} \hat{\theta} &= \gamma\xi - \lambda\ln(x_4). \\ \dot{\xi} &= \frac{\lambda}{\gamma C_2}\left(ux_2 - \hat{\theta}x_4\right)\frac{1}{x_4} \end{aligned}$$

To incorporate the estimator in the PI controller we define the new output

$$\tilde{y} = -\sqrt{Ev_d}x_1 - v_d x_2 + \hat{\theta}\left(\frac{v_d^2}{E}x_3 + v_d\sqrt{\frac{v_d}{E}}x_4\right) \tag{4}$$

It is important to note that in all estimators the only system parameter that is required is C_2. The PI control block diagram is depicted in Fig. 2, while in Fig. 3 we detail the implementation of every estimator.

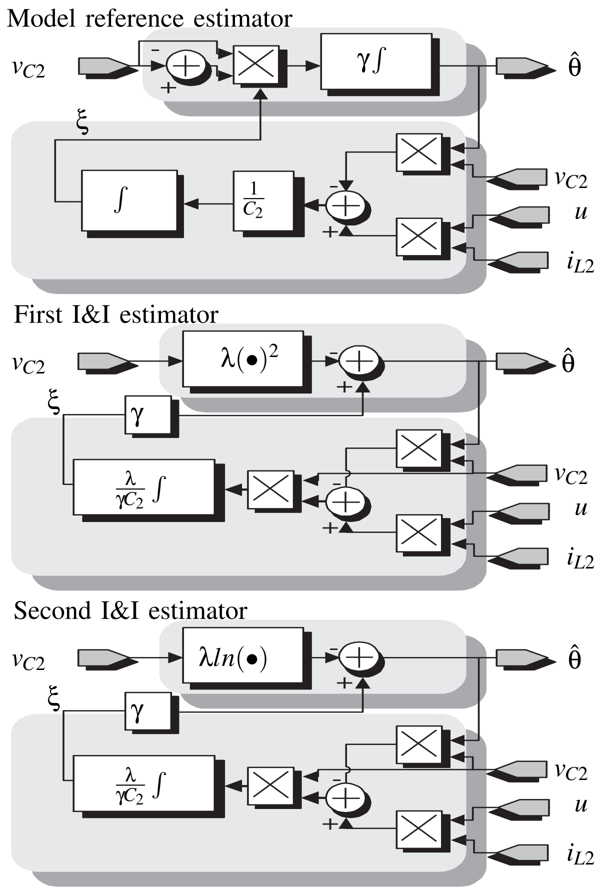


Figure 3: The tree proposed parameter estimators

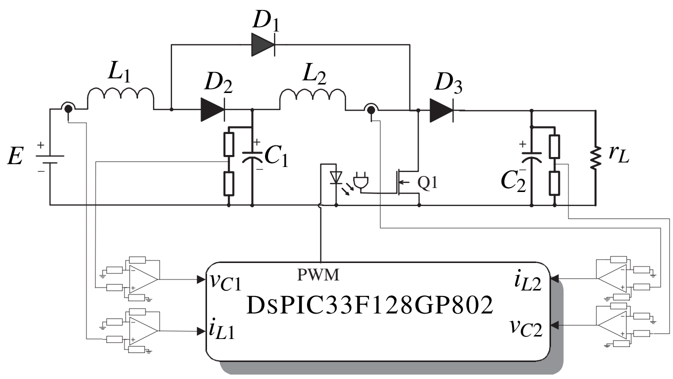


Figure 4: Schematic of the quadratic converter board

IV. Description of the experimental testbench

The experimental card was assembled using low cost commercial electronic elements placed on a card designed in the laboratory. In Fig. 4 we show the diagram of the experimental set-up, consisting of the quadratic circuit card that receives control signals from the Digital Signal Processor (DSP) DsPIC33F128GP802 from Microchip. inc, which is a low cost DSP (3.82 dollars as 2009).

In Fig. 4 we show the main card which is formed by the quadratic converter, a PWM circuit, and some signal conditioners.

The DSP card acquires, using an analog to digital converter integrated in the chip, the capacitors voltages and inductors currents signals previously conditioned from the quadratic converter. Two DC power supplies are necessary to operate the whole system, one to provide energy to the boost system (denoted as power supply E), and the other one to feed the electronic parts of the card.

Two current sensors together with two current to voltage converters are introduced to measure the inductors currents i_{L1} and i_{L2} (in this way, in the form of a voltage signal we

Element	Value
Mosfet	IRFS38N20D
Diodes Schottky	MBRB20200CT
Current Sensor	HSX-50
Capacitances	CKG57NX7R2A475M

Table I: Electronic components used in the experimental setup.

	K_p	K_i	λ	γ
Model reference estimator	$1.5*10^{-1}$	100	$1*10^{5}$	$1*10^{-4}$
First I&I estimator	$1.5*10^{-1}$	100	$2*10^{-6}$	$1*10^{3}$
Second I&I estimator	$1.5*10^{-1}$	100	$1*10^{-3}$	$2*10^{-3}$

Table II: Tuning parameters of the different controllers for the quadratic boost.

can feed it into the DSP card to be used in the control law). In the case of the capacitors voltages v_{C1} and v_{C2}, we put two voltages divisors so we can reduce the level of those signals in such a way that its final value is always in the range [0,3.3] Volts by rail to rail amplifiers.

The following remarks concerning the practical implementation are in order:

- Our objective is to assess the dynamic performance of the proposed PI controller, together with the proposed estimators.
- We have chosen the inductor and the capacitor the experimental circuit close to the values calculated for the selected switched frequency (100KHz) and the load power of 50 W. With those values, given for a typical commercial application, a significant ripple is present. This ripple, unfortunately, tends to mask the transient performance differences between the various controllers.
- Aiming at a practical solution, the controller implementations are carried out with a low cost DSP. This low cost DSP runs at 40 MIPS (Millions of instruction per second) at 16 bits per instruction. Due the values of the control parameters, the 16 bit resolution is not enough, for that reason the control algorithm is running in 32 bits. To compensate for the performance degradation the program is entirely written in assembler code, completes the algorithm in 10 μs and and important is continuous controller is approximated by a discrete-time controller at a fast sampling time.

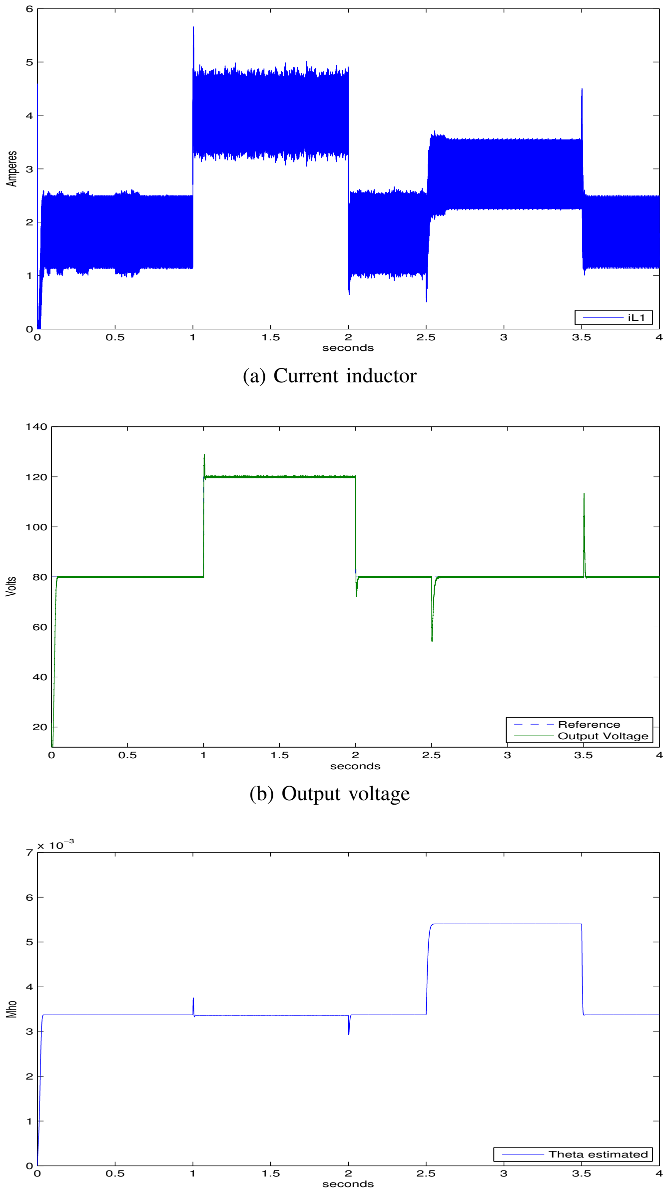
V. Experimental Results

The quadratic converter is composed by two inductors, two capacitors, a resistive charge, three diodes and a switch, the latter implemented by interconnecting a MOSFET transistor and two rapid diodes in a suitable manner, all these elements fed by a DC power supply. See Table I. The values of its elements are taken as $E = 12V$, $L_1 = 53\mu H$, $L_2 = 231\mu H$, $C_1 = 4.7\mu F$, $C_2 = 4.7\mu F$ for all control schemes.

The PI control law described in the previous section has been implemented together with the proposed estimators in the quadratic circuit card. Their performance is compared with the following tests.

- A voltage reference step change from $v_d = 80V$, to $v_d = 120V$
- A change in the load resistance, from $r_L = 330\Omega$ to $r_L = 198\Omega$

The gains of the PI controller and the tuning parameters γ and λ of the estimators are given in Table II.

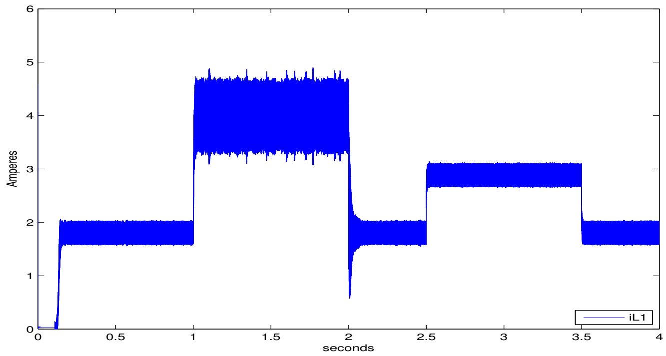


(a) Current inductor

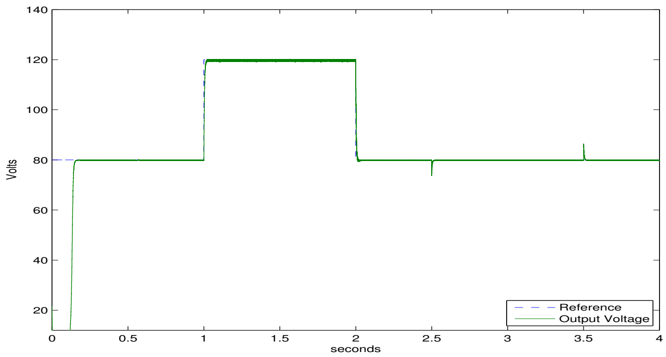
(b) Output voltage

(c) Theta estimation

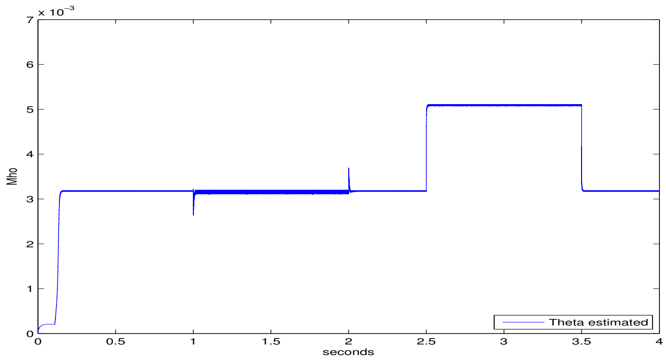
Figure 5: Model Reference experimental results.



(a) Current inductor

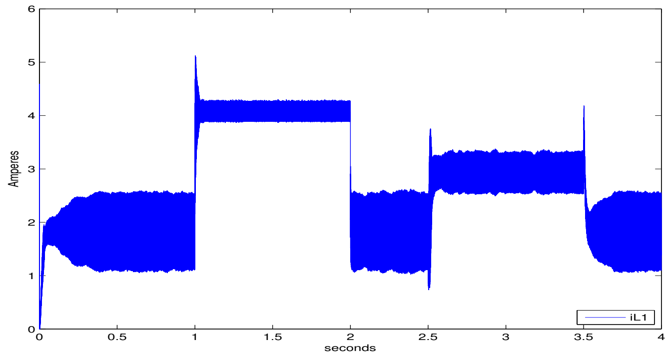


(b) Output voltage

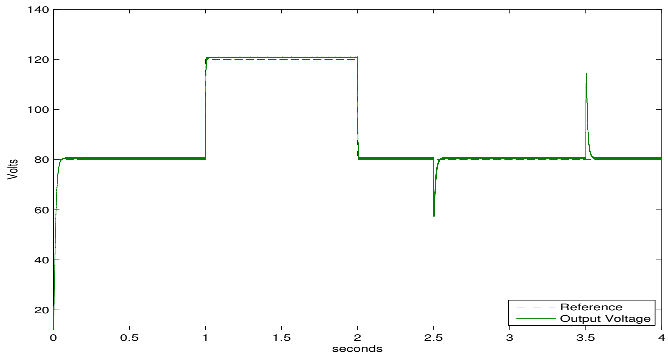


(c) Theta estimation

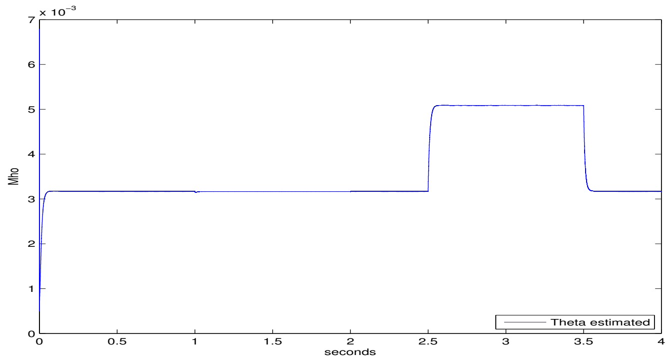
Figure 6: I&I 1 experimental results.



(a) Current inductor



(b) Output voltage



(c) Theta estimation

Figure 7: I&I 2 experimental results.

In Figs. 5a—7c we show the behavior of the state variables and the estimation error for the adaptive PI with the various estimators. As shown in the plots, parameter convergence is achieved in all cases. Due to the reduction of the amplitude in the peaks of v_{C2}, less noisy inductor current i_{L1} and quicker parameter convergence observed in the graphs, we conclude that the I&I 1 controller is more efficient than I&I 2 and this, in its turn, better than the model reference one.

VI. Conclusion

This paper presents a robust adaptive PI controller for a quadratic converter with unknown load resistance. As shown in [2], the controller ensures global asymptotic stabilization for the original nonlinear system—i.e., neither the design nor the analysis rely on linear approximations. Instead, we exploit the fundamental property of incremental passivity of power converters, first reported in [5], and later extended in [2]—see also [3] for a result for more general nonlinear systems. The performance of the PI with three different estimators is studied in an experimental setup of 50 W.

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