An overlapping non-matching grid mortar element method for Maxwell's equations

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***Abstract*—In this paper, a new finite element mortar approach with moving non-matching overlapping grids is introduced. The bidirectional transfer of information between the fixed and moving subdomains is realized for each new position of the moving part. Two numerical examples are presented to support the theory. The first, an electrostatic problem with known solution, to state the optimality of the method. The second, an eddy current nondestructive testing application, to underline the flexibility and efficiency of the proposed approach.**

***Index Terms*—Finite element methods, interface conditions, Maxwell equations, nondestructive testing, overlapping meshes.**

I. Introduction

The modelization in eddy current (EC) nondestructive testing (NDT) aims at reproducing the interaction between a sensor and a conductor in order to localize possible defects in the latter. The finite element method (FEM) is frequently used in this context as well suited to treat problems with complex geometries while keeping a simplicity in the implementation. However, in NDT, the modelization has to be realized for different positions of the sensor, thus requiring a global remeshing of the domain. In the last decades, different techniques to take into account the movement of a sensor without having to remesh the whole computational domain [1]-[4] have been studied. The mortar element method [5] (MEM), a variational non-conforming domain decomposition approach, offers attractive advantages in terms of flexibility and accuracy. In its original version for non-overlapping subdomains, the information is transferred through the skeleton of the decomposition by means of a suitable L^2-projection of the field trace [6], [7]. Recently, a MEM with overlapping subdomains but unidirectional information transfert from the master subdomain to the slave one has been proposed [8]. This variant of the MEM is now suitably modified to deal with bidirectional information transfert between overlapping subdomains. The paper is organized as follows: the EC problem is presented in its weak domain decomposition version. The new overlapping non-matching grid MEM is then described. The accuracy of this new approach is tested on an electrostatic problem, for which the analytical solution is well-known. The method is then applied to solve an EC NDT problem to show its flexibility.

II. Magnetodynamic formulation

The time-harmonic EC equations in a domain Ω read:

$$\mathbf{curl}\,\mathbf{H} = \mathbf{J}, \quad \mathbf{curl}\,\mathbf{E} = -i\omega\mathbf{B}, \quad \mathrm{div}\,\mathbf{B} = 0 \tag{1}$$

where $\mathbf{H}$, $\mathbf{B}$, $\mathbf{J}$ and $\mathbf{E}$ denote the magnetic field, the magnetic flux density, the current density and the electric field. The constitutive laws $\mathbf{B} = \mu\mathbf{H}$, $\mathbf{J} = \mathbf{J_0} + \sigma\mathbf{E}$, with the magnetic permeability μ, the electric conductivity σ and the source current $\mathbf{J_0}$ distributed in Ω as shown in Fig.1 left, are added together with boundary conditions to close the system (1). In this work, a 2D case is examined, where $\mathbf{B}$ lies in the $z = 0$ plane. The magnetic vector potential $\mathbf{A}$ defined by $\mathbf{B} = \mathbf{curl}\,\mathbf{A}$ is thus perpendicular to the $z = 0$ plane and A is its only unknown z-component.

Let us consider the domain Ω where an EC problem (1) is solved. In a domain decomposition approach, we separate the fixed and the moving subdomains in Ω. The moving part, Ω_M, contains the coil supporting $\mathbf{J_0}$ with a layer of air, whereas the fixed part, $\Omega\backslash\Omega_M$, includes some conducting and non-conducting parts (Fig. 1 right). In term of scalar potential A, the problem (1) is reduced to solve magnetostatic or magnetodynamic equations in each subdomain with $A - A_M = 0$, $\mu_M^{-1}\partial_n A_M - \mu^{-1}\partial_n A = 0$ as interface conditions at $\Gamma = \partial\Omega_M$ and homogeneous Dirichlet conditions at $\partial\Omega$.

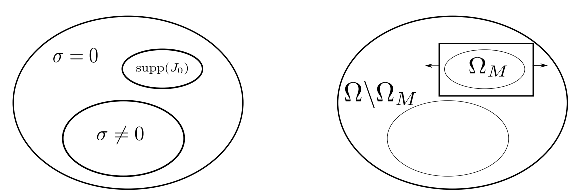

Figure 1: Domain Ω for an EC case (left). Domain decomposition into a moving Ω_M and a fixed $\Omega\backslash\Omega_M$ subdomain (right).

The associated variational problem reads:
Find $(A, A_M) \in H^1_{A_M}(\Omega\backslash\Omega_M) \times H^1_A(\Omega_M)$ such as:

$$\int_{\Omega\backslash\Omega_M} i\sigma\omega AA' + \int_{\Omega\backslash\Omega_M} \mu^{-1}\nabla A.\nabla A' = 0, \quad \forall A' \in H^1_{A_M}(\Omega\backslash\Omega_M)$$
$$\int_{\Omega_M} \mu^{-1}\nabla A_M.\nabla A'_M = \int_{\mathrm{supp}J_0} J_0 A'_M \quad \forall A'_M \in H^1_A(\Omega_M) \tag{2}$$

where $H^1_{A_M}(\Omega\backslash\Omega_M) = \{A \in H^1(\Omega\backslash\Omega_M),\ A = A_M \text{ on } \Gamma\}$ and $H^1_A(\Omega_M) = \{A_M \in H^1(\Omega_M),\ A_M = A \text{ on } \Gamma\}$. According to the Lax-Milgram theorem, there is a unique solution for (2).
The continuous case gives $A|_\Gamma = A_M|_\Gamma$ but the main difficulty arises with the discrete case.

III. Mortar Element Method

Two triangulations are applied depending on the considered subdomains. These discretizations are non matching grids and completely independent in the overlapping region.
The exchange from Ω to Ω_M is realized on the interface Γ [8]. The information from moving to fixed subdomains using an interface chosen as a boundary (called γ) of the elements of Ω overlapped by Γ such as:

$$CA_{M|\Gamma} = DA \quad (3)$$
$$EA_{|\gamma} = HA_M \quad (4)$$

according to the mortar method. The coupling matrices are described, on the edges e of the concerned interface, as:

$$C(i,j) = \int_{e\in\Gamma} \psi_j\psi_i, \quad D(i,k) = \int_{e\in\Gamma} \phi_k\psi_i$$
$$E(i,j) = \int_{e\in\gamma} \phi_j\phi_i, \quad H(i,k) = \int_{e\in\gamma} \psi_k\phi_i$$

where the functions ϕ_i and ψ_i are defined on the discretizations of Ω and Ω_M, respectively. The matrices C and E can be easily computed since both basis functions are defined with respect to the same mesh. On the contrary, D and H concern discrete functions living on different meshes. To solve this problem, quadrature formulas are implemented.
In order to solve a single algebraic system, the conditions (3)-(4) are imposed, on the global matrix, with the help of the method of Lagrange Multipliers [9].

IV. Electrostatic case

In order to evaluate the accuracy of the method, a simple problem is proposed. An uniformly charged cylinder is considered. The moving domain contains the volumic charges and a box of air Fig.2.As the analytical solution is well-known, numerical tests show the accuracy and the continuity of the solution between the subdomains.

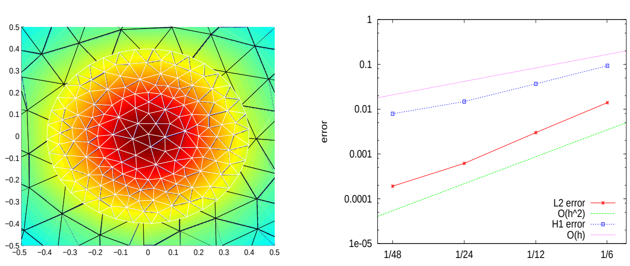

Figure 2: Focus on Ω_M of the electric scalar potential and convergence rates in the fixed domain.

V. NDT case

A 2D EC NDT case is proposed. The coil and a layer of air are considered as the moving domain Ω_M as shown in Fig.3 (left). The computed flux density is given in Fig.3 (right).

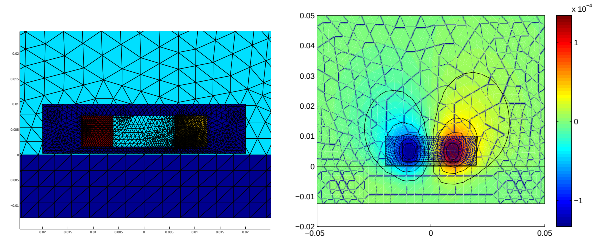

Figure 3: Focus on moving subdomain (left). Magnetic flux density (right).

The values obtained on the two overlapping meshes are compared to the Matlab's PDE tool results on a single mesh. The magnetic flux is calculated in the coil and the results show a difference of 3%. Henceforth, it is possible to determine the flux for different positions of the coil without remeshing the global domain.

VI. Conclusion

The variant of the MEM presented in this paper has the novelty to allow overlapping subdomains with bidirectional exchanges. In the full paper, the calculation of the coupling matrix is explained, the quadrature formula are detailed and the final linear system is defined. Additional results will be provided and the method efficiency discussed.

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