# Numerical method with overlapping non-matching grids to take into account the displacement for eddy current problems

A. Christophe[1], L. Santandrea[1], F. Rapetti[2], G. Krebs[1], Y. Le Bihan[1]
[1] Lab. de Génie Electrique de Paris (LGEP), UMR 8507 CNRS, Univ. Paris-Sud, UPMC Univ. Paris 06, 11 rue Joliot-Curie, Plateau de Moulon, 91192 Gif-sur-Yvette cedex, France
alexandra.christophe@lgep.supelec.fr, yann.le-bihan@lgep.supelec.fr
[2] Univ. Nice Sophia Antipolis, CNRS, Lab. J.-A Dieudonné, UMR 7351, 06100 Nice, France.

***Abstract*—In this paper, a new mortar element approach with overlapping is proposed with bidirectional transfer of information combined with an edge element discretization. The accuracy is examinated and the new method is exposed for a simple 2D eddy current problem.**

***Index Terms*—Finite element method, edge element approximation, overlapping meshes, eddy current problem.**

## I. Introduction

The mortar element method (MEM) [1], a variational nonconforming domain decomposition approach, offers attractive advantages in terms of flexibility and accuracy to take into account the displacement in several applications: more specifically in eddy current non-destructive testing. The main reasons lie in the fact that signals from the sensors must be evaluated for several positions of the defect or of sensor itself. Recently, a MEM with overlapping subdomains but unidirectional information transfert has been proposed [3]. The method of the present work is a new MEM developed to deal with bidirectional information transfer between overlapping subdomains under a discretization with edge elements.

## II. Magnetodynamic formulation

A global domain $\Omega$ separated between $\Omega\backslash\Omega_M$ and $\Omega_M$ subdomains ($\Gamma = \partial\Omega_M$) is considered. The problem is reduced to solve the eddy current equations in 2D in each subdomain with magnetic potentials $\mathbf{A} \subset \Omega\backslash\Omega_M$, $\mathbf{A_M} \subset \Omega_M$ and homogeneous Dirichlet conditions on $\partial\Omega$. The associated variational formulation is given by:
Find $(\mathbf{A}, \mathbf{A_M}) \in H_{\{\mathbf{0},\mathbf{A_M}\}}(\mathbf{curl}, \Omega\backslash\Omega_M) \times H_{\mathbf{A}}(\mathbf{curl}, \Omega_M)$ such as: $\forall \mathbf{A}' \in H_{\{\mathbf{0},\mathbf{A_M}\}}(\mathbf{curl}, \Omega\backslash\Omega_M)$ and $\forall \mathbf{A}'_{\mathbf{M}} \in H_{\mathbf{A}}(\mathbf{curl}, \Omega_M)$

$$\begin{aligned}(i\sigma\omega\mathbf{A}, \mathbf{A}')_{L^2(\Omega\backslash\Omega_M)} + (\mu^{-1}\,\mathbf{curl}\,\mathbf{A}, \mathbf{curl}\,\mathbf{A}')_{L^2(\Omega\backslash\Omega_M)} &= (J_0, \mathbf{A}')_{L^2(\Omega\backslash\Omega_M)},\\ (i\sigma_M\omega_M\mathbf{A_M}, \mathbf{A}'_{\mathbf{M}})_{L^2(\Omega_M)} + (\mu^{-1}\,\mathbf{curl}\,\mathbf{A_M}, \mathbf{curl}\,\mathbf{A}'_{\mathbf{M}})_{L^2(\Omega_M)} &= (J_0, \mathbf{A}'_{\mathbf{M}})_{L^2(\Omega_M)}\end{aligned} \quad (1)$$

where $H_{\mathbf{A}}(\mathbf{curl}, \Omega_M) = \{\mathbf{A_M} \in H(\mathbf{curl}, \Omega_M) :\ \mathbf{A_M}.\tau_\Gamma = \mathbf{A}.\tau_\Gamma\}$ and $H_{\{\mathbf{0},\mathbf{A_M}\}}(\mathbf{curl}, \Omega\backslash\Omega_M) = \{\mathbf{A} \in H(\mathbf{curl}, \Omega\backslash\Omega_M) :$ $\mathbf{A}.\tau_{\partial\Omega} = \mathbf{0},\ \mathbf{A}.\tau_\Gamma = \mathbf{A_M}.\tau_\Gamma\}$. $(.,.)_{L^2(.)}$ denote the $L^2$-norm.
The continuous problem yields $(\mathbf{A}.\tau)|_\Gamma = (\mathbf{A_M}.\tau)|_\Gamma$, then we focus our attention on the discrete case.

## III. Mortar method

Two discretizations in space, $\mathfrak{H}$ and $\mathfrak{h}$ , depending on each subdomain, $\Omega$ and $\Omega_M$, are considered. They are non-matching grids and completely independent in the overlapping region. The transfer of information is realized on well-chosen interfaces, $\Gamma_\mathfrak{h}$ (from $\Omega$ to $\Omega_M$) and $\gamma_\mathfrak{H}$ the boundary of the elements of $\Omega$ overlapped by $\Gamma_\mathfrak{h}$(form $\Omega_M$ to $\Omega$), reads:

$$C\mathbf{A}_{\mathbf{M}|\Gamma_\mathfrak{h}} = D\mathbf{A} \qquad E\mathbf{A}_{|\gamma_\mathfrak{H}} = H\mathbf{A_M} \quad (2)$$

The coupling matrices issued from (2) are described, on the edges $e$ of the concerned interface, as:

$$C(e,j) = \int_{e\in\Gamma_\mathfrak{h}} (\mathbf{w_e^M}.\tau_e)(\mathbf{w_j^M}.\tau_e),\ D(e,k) = \int_{e\in\Gamma_\mathfrak{h}} (\mathbf{w_e^M}.\tau_e)(\mathbf{w_k^F}.\tau_e),$$
$$E(e,j) = \int_{e\in\gamma_\mathfrak{H}} (\mathbf{w_e^F}.\tau_e)(\mathbf{w_j^F}.\tau_e),\ H(e,k) = \int_{e\in\gamma_\mathfrak{H}} (\mathbf{w_e^F}.\tau_e)(\mathbf{w_k^M}.\tau_e)$$

where $\mathbf{w_e^F}$ and $\mathbf{w_e^M}$ are the functions of the Whitney edge element space which are defined in the discretizations of $\Omega$ and $\Omega_M$. The main difficulty is to compute the matrices $D$ and $H$ which concern discrete functions living on different meshes. To solve this problem, quadrature formulas are implemented. In order to solve a single algebraic system, the conditions (2) are imposed, on the global matrix, with the help of a Lagrange multipliers method.
The accuracy of the method is evaluated for two-dimensional electric equation. As the analytical solution is well-known, numerical tests show the accuracy and the optimality of the solution between the subdomains: the orders of convergence in $L^2$-norm and $H(\mathbf{curl})$-norm are maintained. Moreover, calculations for an eddy current problem will be presented and comparaisons with standard software are realised. (Fig. 1)

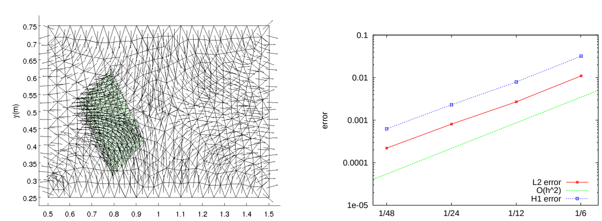


Fig. 1. Electric field distribution (left). Convergence rates for the global solution(right)

In the extended paper, the calculation of the coupling matrices, the quadrature formulas and the final linear system will be described.

## References

[1] C. Bernardi, Y. Maday, A. Patera, *A new non-Conforming approach to domain decomposition: the mortar element method,* Seminaire XI du College de France, Brezis & Lions eds., in Nonlinear partial differential equations and their applications, Pitman, pp. 13-51, 1994.
[2] F. Ben Belgacem, *The Mortar Finite Element Method with Lagrange Multipliers,* Numer. Math., vol 84, pp. 173-197, 1999.
[3] F. Rapetti, *An Overlapping Mortar Element Approach to Coupled Magneto-Mechanical Problems,* Math. Comput. Simul., vol.80, 2010.